

# TEXAS A&M HIGH PERFORMANCE RESEARCH COMPUTING

## Introduction to Quantum Mechanics Short Course

Lecture 5 of 6



TEXAS A&M UNIVERSITY  
Division of Research



## Excited State Methods

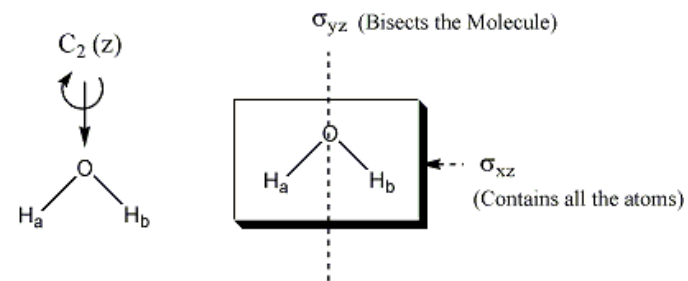
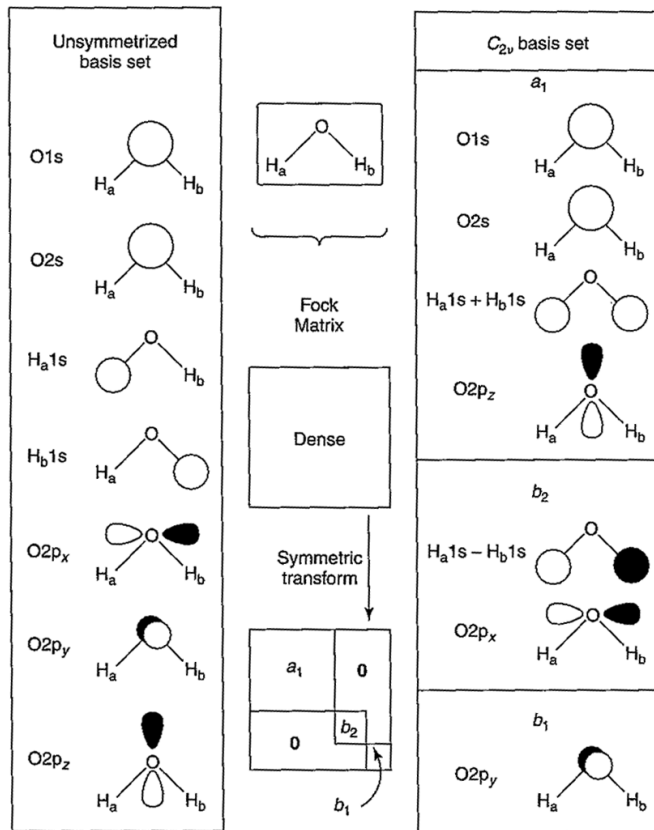
If the excited state has a different spin or symmetry, you can perform a Single Point Energy (SPE) or Geometry Optimization using the correct spin and/or orbital occupation.

$O_2$  triplet ground state  
singlet excited state

$O_s C_2^+$  in  $C_{2v}$  symmetry:  
 ${}^2A_1, {}^2A_2, {}^2B_1, \text{ or } {}^2B_2$

$C_{2v}$	E	$C_2(z)$	$\sigma_v(xz)$	$\sigma_v(yz)$	linear functions, rotations	quadratic functions	cubic functions
$A_1$	+1	+1	+1	+1	z	$x^2, y^2, z^2$	$z^3, x^2z, y^2z$
$A_2$	+1	+1	-1	-1	$R_z$	xy	xyz
$B_1$	+1	-1	+1	-1	x, $R_y$	xz	$xz^2, x^3, xy^2$
$B_2$	+1	-1	-1	+1	y, $R_x$	yz	$yz^2, y^3, x^2y$

# Symmetry and Excited States



$C_{2v}$	E	$C_2$	$\sigma_v(yz)$	$\sigma_v(xz)$
$A_1$	1	1	1	1
$A_2$	1	1	-1	-1
$B_1$	1	-1	1	-1
$B_2$	1	-1	-1	1
$a_1xb_1$	1x1	1x(-1)	1x(1)	1x(-1)
$b_1$	1	-1	1	-1
$b_1xb_2$	1x1	-1x(-1)	1x(-1)	-1x(1)
$a_2$	1	1	-1	-1

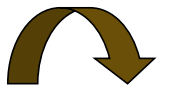
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Orbital symmetries:
Alpha Orbitals:
  Occupied (A1) (A1) (B2) (B1) (A1) (A1)
  Virtual (B2)
Beta Orbitals:
  Occupied (A1) (A1) (B2) (A1)
  Virtual (B1) (A1) (B2)
The electronic state is 3-B1.
Alpha occ. eigenvalues -- -20.43372 -1.43457 -0.70542 -0.63599 -0.62365
Alpha occ. eigenvalues -- 0.02368
Alpha virt. eigenvalues -- 0.56142
Beta occ. eigenvalues -- -20.38194 -1.19468 -0.62529 -0.50668
Beta virt. eigenvalues -- 0.27727 0.63457 0.70966
Alpha Molecular Orbital Coefficients
      1      2      3      4      5
EIGENVALUES -- (A1)--O (A1)--O (B2)--O (B1)--O (A1)--O
1 1 O 1S      -20.43372 -1.43457 -0.70542 -0.63599 -0.62365
2 2 S          0.99408 -0.24869 0.00000 0.00000 -0.07293
3 2PX         0.02635 0.93457 0.00000 0.00000 0.36237
4 2PY         0.00000 0.00000 0.00000 1.00000 0.00000
5 2PZ        -0.00306 -0.13811 0.00000 0.00000 0.87907
6 2 H 1S      -0.00560 0.08032 0.41335 0.00000 -0.18056
7 3 H 1S      -0.00560 0.08032 -0.41335 0.00000 -0.18056
      6      7
EIGENVALUES -- (A1)--O (B2)--V
1 1 O 1S      0.02368 0.56142
2 2 S      -0.80891 0.00000
3 2PX      0.00000 0.00000
4 2PY      0.00000 0.95591
5 2PZ      0.58523 0.00000
6 2 H 1S    0.81596 -0.81581
7 3 H 1S    0.81596 0.81581
Beta Molecular Orbital Coefficients.
      1      2      3      4      5
EIGENVALUES -- (A1)--O (A1)--O (B2)--O (A1)--O (B1)--V
1 1 O 1S      -20.38194 -1.19468 -0.62529 -0.50668 0.27727
2 2 S          0.99466 -0.24048 0.00000 -0.06484 0.00000
3 2PX         0.02379 0.88678 0.00000 0.35254 0.00000
4 2PY         0.00000 0.00000 0.00000 0.00000 1.00000
5 2PZ         0.00000 0.00000 0.72027 0.00000 0.00000
6 2 H 1S      -0.00255 -0.03603 0.00000 0.85179 0.00000
7 3 H 1S      -0.00495 0.13466 0.33814 -0.22974 0.00000
Beta virt. eigenvalues -- 0.27727 0.63457 0.70966

```

$$b_1 \times a_1 \rightarrow 3B_1$$

If we want the  $3A_1$  state, we will need to swap  $\beta$  orbitals 4 and 5 using the guess=(read,alter) keyword



```

%chk=h2o.chk
#p hf/sto-3g pop=full guess=(read,alter)
bl
title
bl
0 3
O
H 1 bl
H 1 bl 2 a1
bl
bl 1.0
a1 109.5
bl
bl ← Alpha orbital swap section
4 5 ← Beta orbital swap section
bl

```

Alpha Molecular Orbital Coefficients

		1	2	3	4	5
		(A1)--O	(A1)--O	(A1)--O	(B2)--O	(B1)--O
EIGENVALUES	--	-20.35050	-1.37970	-0.68279	-0.68060	-0.49934
1 1	O 1S	0.99410	-0.24739	-0.06231	0.00000	0.00000
2	2S	0.02615	0.92150	0.32104	0.00000	0.00000
3	2PX	0.00000	0.00000	0.00000	0.00000	1.00000
4	2PY	0.00000	0.00000	0.00000	0.59599	0.00000
5	2PZ	-0.00220	-0.07595	0.88247	0.00000	0.00000
6 2	H 1S	-0.00541	0.10043	-0.18707	0.44725	0.00000
7 3	H 1S	-0.00541	0.10043	-0.18707	-0.44725	0.00000

Beta Molecular Orbital Coefficients.

		1	2
		(A1)--O	(B2)--V
EIGENVALUES	--	0.03115	0.57703
1 1	O 1S	-0.12783	0.00000
2	2S	0.84074	0.00000
3	2PX	0.00000	0.00000
4	2PY	0.00000	0.98180
5	2PZ	-0.59145	0.00000
6 2	H 1S	-0.81226	-0.79773
7 3	H 1S	-0.81226	0.79773

Beta Molecular Orbital Coefficients.

		1	2	3	4	5
		(A1)--O	(A1)--O	(B2)--O	(B1)--O	(A1)--V
EIGENVALUES	--	-20.30251	-1.16482	-0.59165	-0.42211	0.15055
1 1	O 1S	0.99450	-0.24181	0.00000	0.00000	-0.09597
2	2S	0.02448	0.89422	0.00000	0.00000	0.53599
3	2PX	0.00000	0.00000	0.00000	1.00000	0.00000
4	2PY	0.00000	0.00000	0.72450	0.00000	0.00000
5	2PZ	-0.00204	-0.06820	0.00000	0.00000	0.70571
6 2	H 1S	-0.00508	0.12513	0.33411	0.00000	-0.38197
7 3	H 1S	-0.00508	0.12513	-0.33411	0.00000	-0.38197

Beta Molecular Orbital Coefficients.

		1	2
		(A1)--V	(B2)--V
EIGENVALUES	--	0.70643	0.74863
1 1	O 1S	0.11385	0.00000
2	2S	-0.75648	0.00000
3	2PX	0.00000	0.00000
4	2PY	0.00000	0.89120
5	2PZ	0.79476	0.00000
6 2	H 1S	0.73709	-0.85134
7 3	H 1S	0.73709	0.85134

No Alpha orbitals switched.  
Pairs of Beta orbitals switched:  
4 5  
Initial guess orbital symmetries:  
Alpha Orbitals:  
Occupied (A1) (A1) (B2) (B1) (A1) (A1)  
Virtual (B2)  
Beta Orbitals:  
Occupied (A1) (A1) (B2) (B1)  
Virtual (A1) (A1) (B2)  
The electronic state of the initial guess is 3-A1.  
Alpha Orbitals:  
Occupied (A1) (A1) (A1) (B2) (B1) (A1)  
Virtual (B2)  
Beta Orbitals:  
Occupied (A1) (A1) (B2) (B1)  
Virtual (A1) (A1) (B2)  
The electronic state is 3-A1.  
Alpha occ. eigenvalues -- -20.35050 -1.37970 -0.68279 -0.68060 -0.49934  
Alpha occ. eigenvalues -- 0.03115  
Alpha virt. eigenvalues -- 0.57703  
Beta occ. eigenvalues -- -20.30251 -1.16482 -0.59165 -0.42211  
Beta virt. eigenvalues -- 0.15055 0.70643 0.74863



# Excited State Methods

**CIS** – Configuration Interaction Singles. CIS gives HF quality results for excited states.

**RPA** – Random Phase Approximation. Similar to CIS but includes double excitations in the calculation of the excited state energies.

**CIS(D)** – Configuration Interaction Singles with 2<sup>nd</sup> order Perturbative correction for the excitation energies.

**Time-Dependent Density Functional Theory (TD-DFT)**



## Excited State Methods

- CIS – Configuration Interaction Singles.
  - CIS gives HF quality results for excited states

TABLE III. Lowest lying singlet states of  $C_6H_6$  at the 6-31+G\* level of theory using various theoretical methods.

Symmetry	CIS	RPA	B3LYP	BPW91	LSDA	Experiment
$B_{2u} (\pi \rightarrow \pi^*)$	6.15	5.96	5.40	5.19	5.31	4.9
$B_{1u} (\pi \rightarrow \pi^*)$	6.31	6.01	6.06	5.93	6.10	6.2
$E_{1g} (\pi \rightarrow 3s)$	7.13	7.12	6.34	6.34	6.36	6.33
$A_{2u} (\pi \rightarrow 3p)$	7.45	7.43	6.84	6.87	6.99	6.93
$E_{2u} (\pi \rightarrow 3p)$	7.75	7.74	6.88	6.85	6.98	6.95
$E_{1u} (\pi \rightarrow \pi^*)$	7.94	7.52	6.96	6.84	6.94	7.0

J. Chem. Phys., Vol. 109, 1998, 8218



# Excited State Methods

- CIS

- Oscillator strengths are proportional to extinction coefficient

Excitation energies and oscillator strengths:

Excited State 1: Singlet-A 6.1546 eV 201.45 nm f=0.0000 <S\*\*2>=0.000  
20 -> 27 0.47187  
20 -> 38 -0.15934  
21 -> 28 0.47187  
21 -> 39 -0.15934

This state for optimization and/or second-order correction.

Copying the excited state density for this state as the 1-particle RhoCI density.

Excited State 2: Singlet-A 6.3056 eV 196.63 nm f=0.0000 <S\*\*2>=0.000  
20 -> 28 -0.46761  
20 -> 39 0.14725  
21 -> 27 0.46763  
21 -> 38 -0.14725

Excited State 3: Singlet-A 7.1275 eV 173.95 nm f=0.0000 <S\*\*2>=0.000  
21 -> 22 0.65758  
21 -> 34 -0.11577  
21 -> 48 0.11247





# Benzene CIS output

- Excitation 1  $\pi \rightarrow \pi^*$

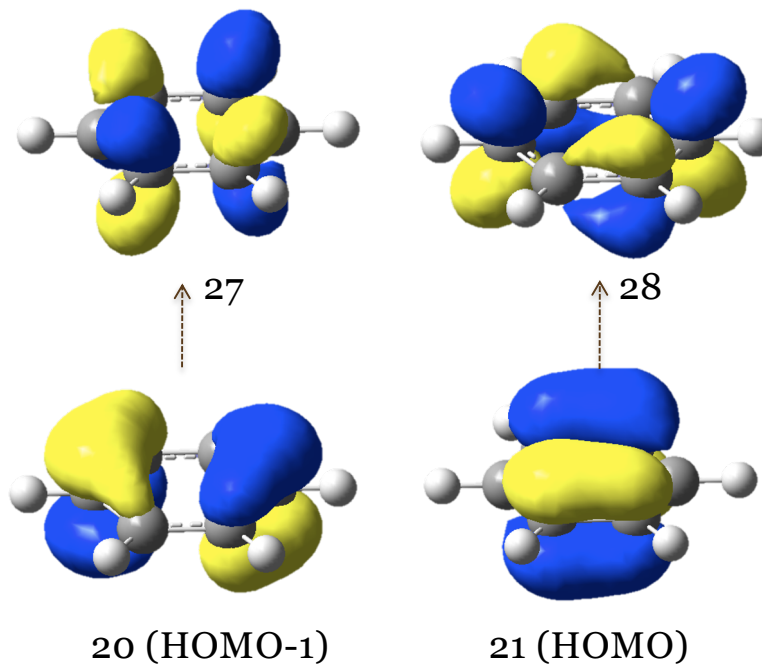
Excited State 1: Singlet-A 6.1546 eV 201.45 nm f=0.0000  $\langle S^{*2} \rangle = 0.0000$

20 -> 27 0.47187

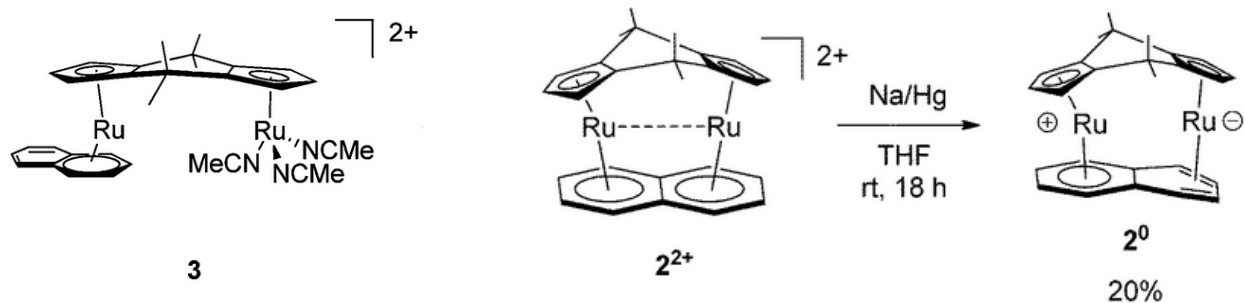
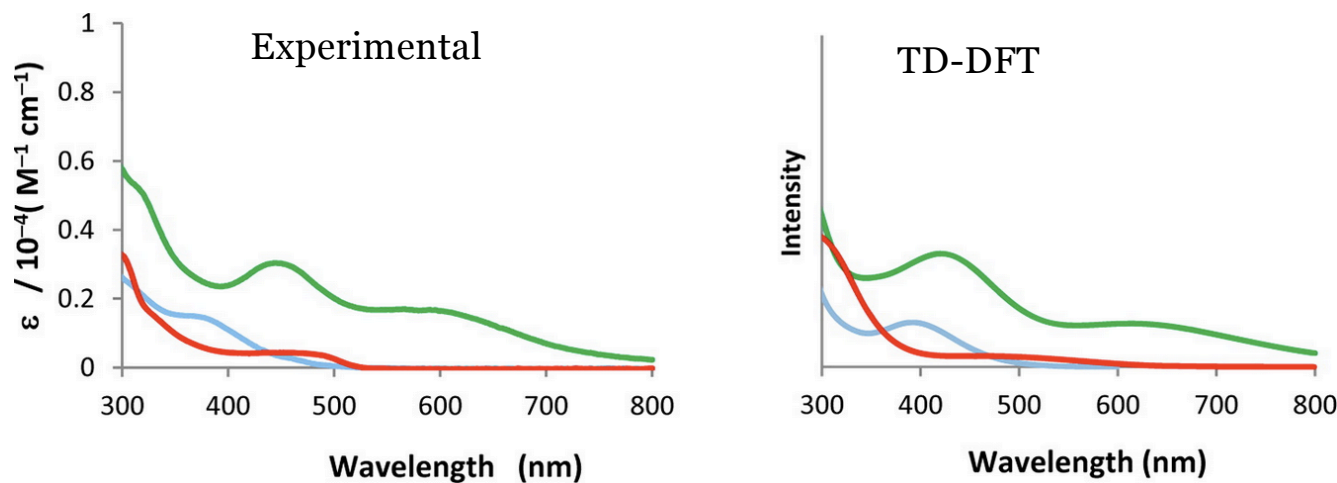
20 -> 38 -0.15934

21 -> 28 0.47187

21 -> 39 -0.15934



# UV/Vis spectra via TD-DFT



## Excited State Methods

**SAC-CI** – Symmetry Adapted Cluster Configuration Interaction

**EOM-CCSD** – Equation of Motion – Coupled Cluster Singles and Doubles.

Multi-Reference Excited State Methods:

**CASSCF** – Complete Active Space Self-Consistent Field. Multi-reference Hartree-Fock.

**CAS-PT<sub>2</sub>** – Complete Active Space Self-Consistent Field with MP<sub>2</sub> corrected energies.

**MRCI** – Multi-Reference Configuration Interaction.



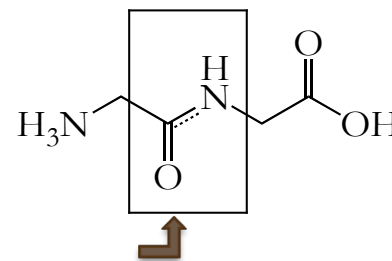
# Semi-empirical Calculations

- Based on HF formulism –  $FC=SC\epsilon$ 
  - Iterative
- core  $e^-$  are not treated explicitly
- each atom is described by 1STO
- $2e^-$  integrals are replaced with parameters ( $V_{ee}$ )
- Parameterized to reproduce Heats of Formation
- Commonly used semi-empirical models
  - AM1 – austin model 1 (Dewar)
  - PM3 – 3<sup>rd</sup> parameterization of MNDO (Stewart)
  - PM7 – 7<sup>th</sup> parameterization of MNDO which includes more transition metals ([http://openmopac.net/PM7\\_accuracy/PM7\\_accuracy.html](http://openmopac.net/PM7_accuracy/PM7_accuracy.html)).
  - ZINDO/S – parameterized to calculate electronic spectra (primarily for organic species)

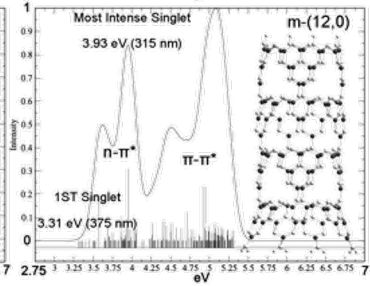
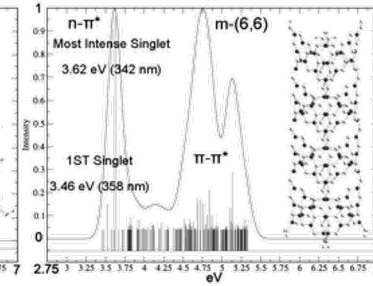
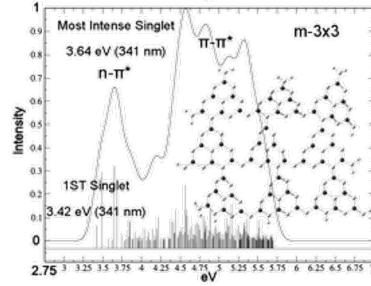
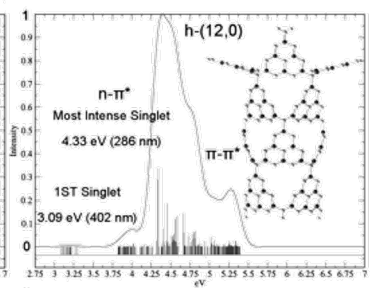
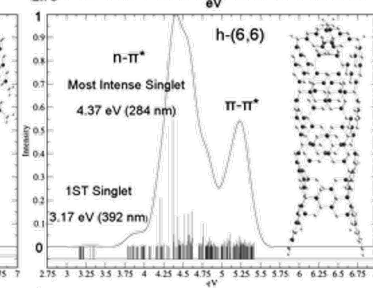
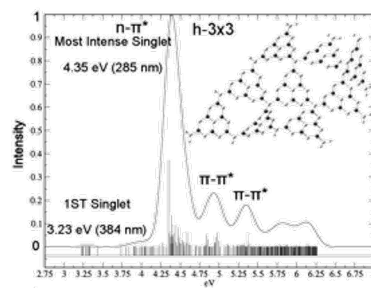
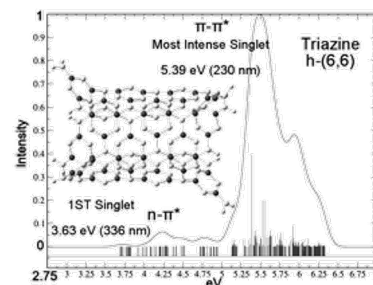
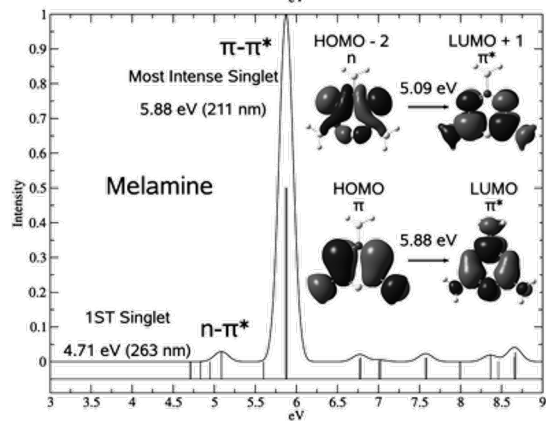
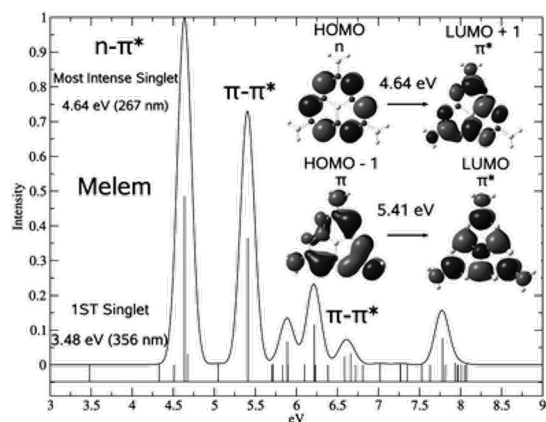


# Semi-Empirical

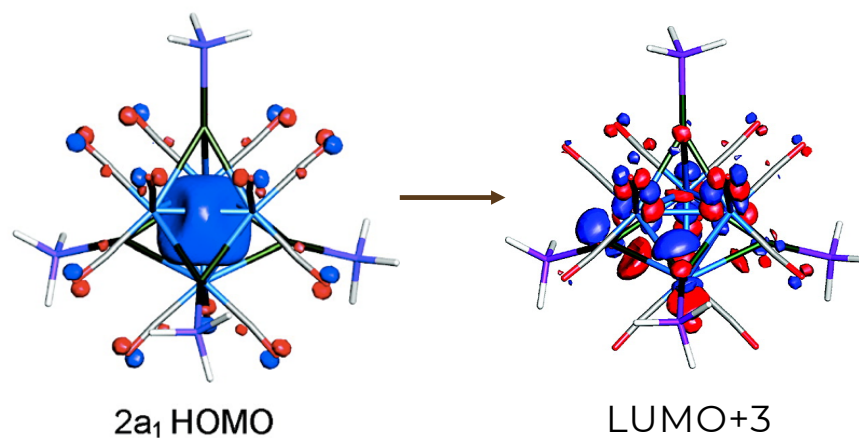
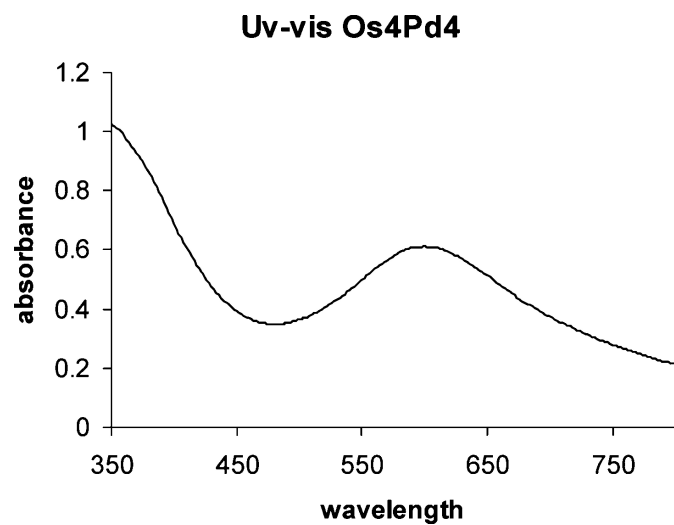
- AM1 – Austin Model 1
  - Heavily used for organic compound containing 1<sup>st</sup> row main group elements (ie C, N, O)
  - Reasonable geometries
  - Poor for hydrogen bonding
  - heavier elements are often represented with MNDO
- PM3 – Parameter Model 3
  - Also heavily used with a wider range of elements available
  - utilizes an extra molecular mechanics term to accurately describe peptide bonds



# ZINDO/S



# Fenske-Hall (FH)



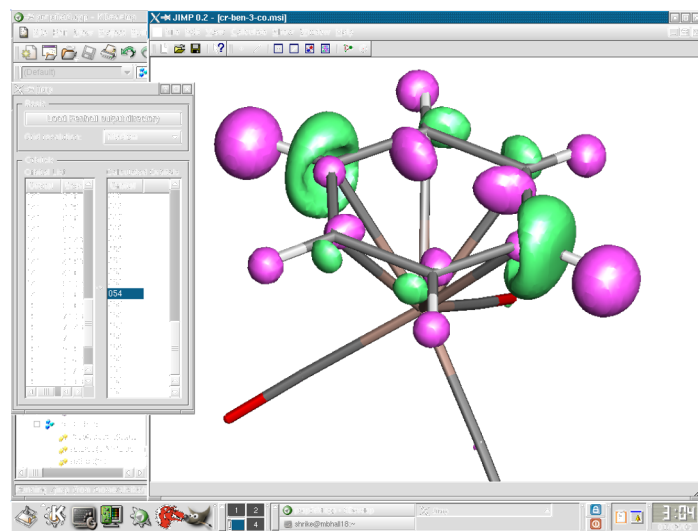
Published in: Richard D. Adams; Erin M. Boswell; Burjor Captain;  
*Organometallics* **2008**, 27, 1169-1173.  
DOI: 10.1021/om701206r  
Copyright © 2008 American Chemical Society

<http://www.chem.tamu.edu/jimp2>



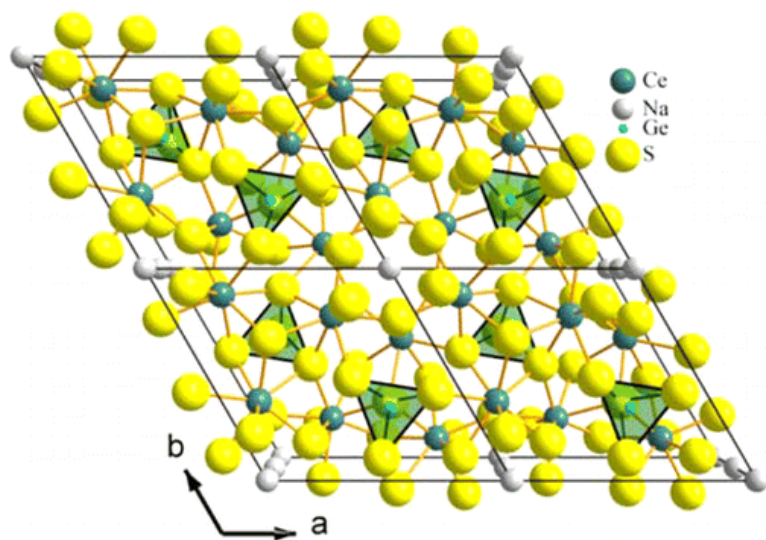
## Fenske-Hall (FH)

- Based on Extended Hückel Theory
- Replaces the diagonal terms ( $H_{\mu\mu}$ ) with an adjustable parameter to account for partial charge changes in molecules.
- Iterative
- Available in Jimp2:  
<http://www.chem.tamu.edu/jimp2>

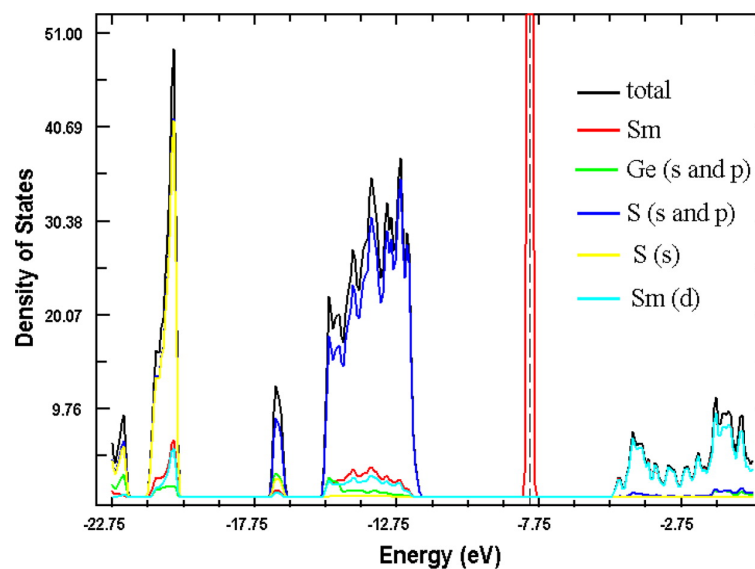




# Extended Hückel Theory (EHT)



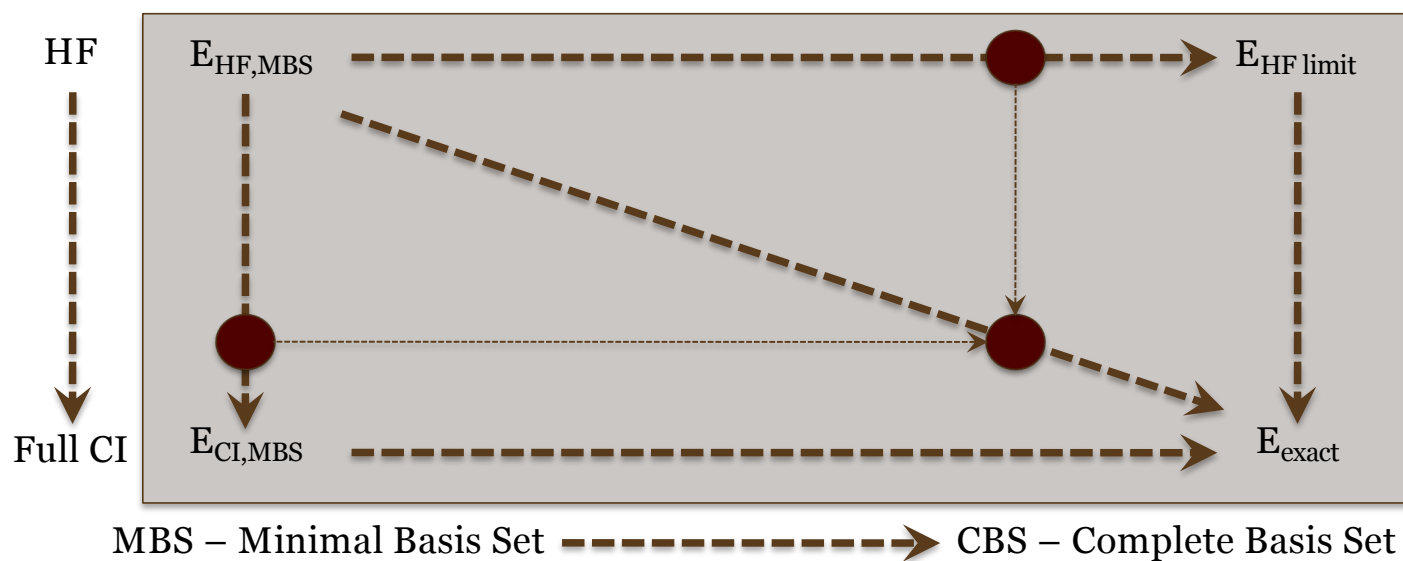
Published in: Amitava Choudhury; Peter K. Dorhout; *Inorg. Chem.* Ahead of Print  
DOI: 10.1021/ic502418s  
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Total and partial (Sm, Ge, and S) DOSs calculated for **III**. The dashed line indicates the position of the Fermi level.

## Composite/Compound/extrapolation Methods

Methods that try to extrapolate to the energy obtained using an infinite basis set and full electron correlation. Designed to accurately calculate thermochemical data.



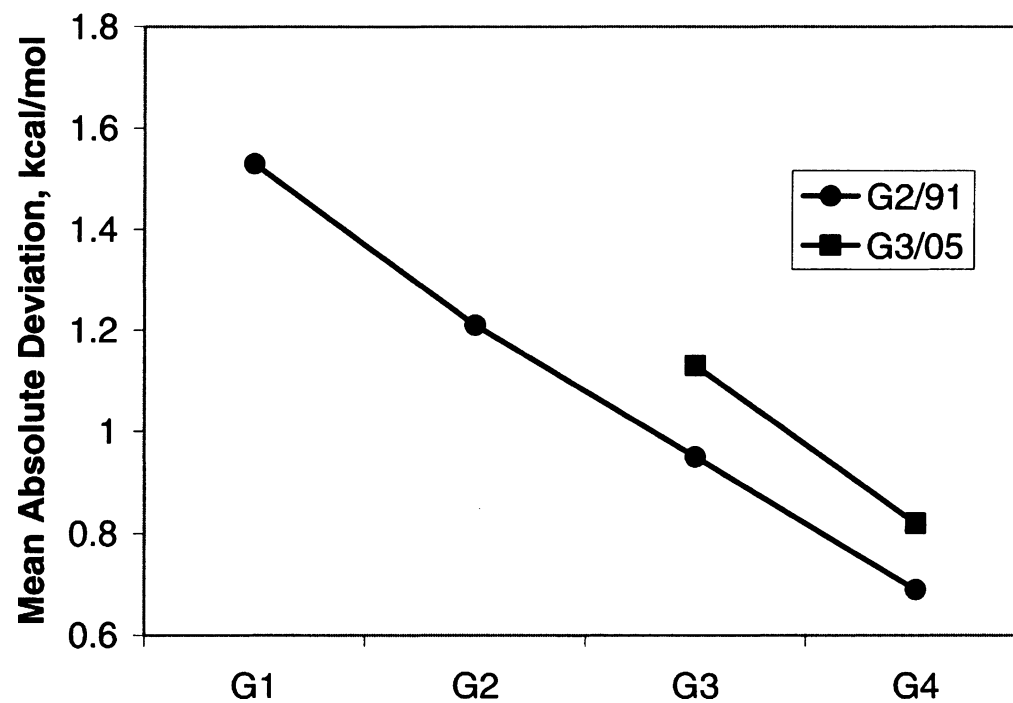
## Composite/Compound/extrapolation Methods

### Basic Method:

- Select a level of theory for the Geometry (optimization)
- Select a level of theory for the zero-point energy (optimize and frequency calculation)
- Select a level of theory for the reference energy calculation (single point energy)
- Perform a variety of single point energy calculations with increasing basis set size and increasing level of theory to estimate basis set effects and dynamic electron correlation.
- Estimate the spin-paired correlation error



## G1-4 Accuracy



J. Chem. Phys. **126**, 084108 (2007)



## Koopmans Theorem

- The energy required to remove an electron from a closed shell atom or molecule is well approximated by the negative of the orbital energy that it is being removed from:

$$1^{\text{st}} \text{ Ionization Energy} \cong -\epsilon_{\text{HOMO}}$$

- Electron Affinity  $\neq \epsilon_{\text{LUMO}}$
- Unoccupied orbitals are not well treated. The coefficient matrix is optimized for the occupied orbitals.

