Introduction to Code Parallelization Using MPI

Ping Luo

TAMU HPRC

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Outline

- Parallel programming models
  - OpenMP for Shared Memory System
  - MPI for Distributed Memory System
  - MPI+OpenMP for hybrid systems
- Layout of an MPI program
- Compiling and running an MPI program
- Basic MPI concepts
  - size, rank, communicator, message, MPI datatype, tag
- Point-to-point communication
- Collective communication
Parallel Computing Systems

We use a processor or a CPU core to refer to the smallest physical processing unit where a program is executed.

- Shared Memory Systems
- Distributed Memory Systems
- Hybrid Systems
Parallel Computing Systems

- **Shared Memory System** – an abstraction to a parallel system where all processors share the same memory subsystem

Example: a node on Ada – an IBM NextScale nx360 M4 dual socket server
Parallel Computing Systems

- **Distributed Memory System** – an abstraction to a parallel system where each processor has its own local memory and the processors don’t share a global memory subsystem.
Parallel Computing Systems

- Hybrid System

![Diagram of a hybrid system with nodes and memory modules connected through an interconnect network.]
Parallel Programming Models

- Mapping from the parallel programming models to the parallel computing systems

- OpenMP: Shared Memory System
- MPI: Distributed Memory System
- MPI+OpenMP: Hybrid System
Ada and Terra

- Ada has 852 nodes and each node has 20 cores.
- Terra has 304 compute nodes and each node has 28 cores.
- OpenMP can only run within a cluster node.
- MPI and Hybrid can run on multiple nodes.
Resources

- Two books: **Using MPI** and **Using MPI2**


- List of all MPI routines: [https://www.mpich.org/static/docs/v3.2/](https://www.mpich.org/static/docs/v3.2/)

- Examples for the course are on Ada: `/general/public/training/mpi/Spring2019`
## Example 1: Hello World

### C

```c
#include <stdlib.h>

int main(int argc, char **argv){
    printf("Hello, world\n");
}
```

### Fortran

```fortran
program hello
implicit none

print *, "Hello, world"

end program hello
```
Example 1: Hello World

C
#include <stdlib.h>

int main(int argc, char **argv){
    printf("Hello, world\n");
}

Fortran
program hello
implicit none

print *, "Hello, world"
end program hello

C
#include <stdlib.h>
#include <mpi.h>

int main(int argc, char **argv){
    MPI_Init(&argc, &argv);
    printf("Hello, world\n");
    MPI_Finalize();
}

Fortran
program hello
use mpi
implicit none

call MPI_INIT(ierr)
print *, "Hello, world"
call MPI_Finalize(ierr)
end program hello
# Layout of an MPI Program

<table>
<thead>
<tr>
<th>C</th>
<th>Fortran</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>#include &lt;mpi.h&gt;</code></td>
<td><code>PROGRAM SAMPLE1</code></td>
</tr>
<tr>
<td><code>int main(int argc, char **argv)</code></td>
<td><code>USE MPI !F90</code></td>
</tr>
<tr>
<td><code>{</code></td>
<td><code>!f77: include “mpif.h”</code></td>
</tr>
<tr>
<td><code>... no mpi calls</code></td>
<td><code>integer ierr</code></td>
</tr>
<tr>
<td><code>MPI_Init(&amp;argc, &amp;argv);</code></td>
<td><code>... no mpi calls</code></td>
</tr>
<tr>
<td>mpi calls happen here</td>
<td><code>CALL MPI_INIT(ierr)</code></td>
</tr>
<tr>
<td></td>
<td>multiple concurrent processes execute at their own pace unless synchronization is applied.</td>
</tr>
<tr>
<td><code>MPI_Finalize();</code></td>
<td>mpi calls happen here</td>
</tr>
<tr>
<td><code>... no mpi calls</code></td>
<td><code>CALL MPI_FINALIZE(ierr)</code></td>
</tr>
<tr>
<td><code>}</code></td>
<td><code>... no mpi calls</code></td>
</tr>
<tr>
<td></td>
<td><code>END PROGRAM SAMPLE1</code></td>
</tr>
</tbody>
</table>
Compiling and Linking MPI Programs

```
module load intel/2017b
mpiicc prog.c [flags] -o prog.exe (C)
mpicpc prog.cpp [flags] -o prog.exe (C++)
mpiifort prog.f [flags] -o prog.exe (Fortran)

(Intel compilers)
mpiicc prog.c [flags] -o prog.exe (C)
mpicxx prog.cpp [flags] -o prog.exe (C++)
mpif90 prog.f [flags] -o prog.exe (Fortran)

(GNU compilers)
```

We will use the Intel MPI and the Intel compilers in ensuing examples.
See HPRC user guide for more information:
https://hprc.tamu.edu/wiki/Ada:Compile:All#MPI_Programs
Running an MPI Program on Login Nodes

- Load the modules first

```bash
module load intel/2017b
```

- Run the mpi program on the login nodes for testing and debugging. **No more than 8 MPI tasks** can be launched at once per our policy.

```bash
mpirun -np n [options] \ prog.exe [prog_args]  
(n is number of MPI tasks and n<=8)
```

- Useful MPI options

```bash
-ppn/-perhost, -hosts, -hostfile, -h
```

- When testing the code on the login nodes, make sure the code works on multiple login nodes.

```bash
mpirun -np 4 -hosts \ login1,login2,login3,\ login4 -ppn 1 mympi.exe
```
### Running an MPI Program in Batch

#### Batch Examples

<table>
<thead>
<tr>
<th>Ada</th>
<th>Terra</th>
</tr>
</thead>
<tbody>
<tr>
<td>#BSUB -J MPIBatchExample</td>
<td>#![/bin/bash</td>
</tr>
<tr>
<td>#BSUB -L /bin/bash</td>
<td>#SBATCH --export=NONE</td>
</tr>
<tr>
<td>#BSUB -W 24:00</td>
<td>#SBATCH --get-user-env=L</td>
</tr>
<tr>
<td>#BSUB -n 40</td>
<td>#SBATCH --job-name=MPIBatchExample</td>
</tr>
<tr>
<td>#BSUB -R &quot;span[ptile=20]&quot;</td>
<td>#SBATCH --time=24:00:00</td>
</tr>
<tr>
<td>#BSUB -R &quot;rusage[mem=2560]&quot;</td>
<td>#SBATCH --ntasks=56</td>
</tr>
<tr>
<td>#BSUB -M 2560</td>
<td>#SBATCH --ntasks-per-node=28</td>
</tr>
<tr>
<td>#BSUB -o MPIBatchExample.%J</td>
<td>#SBATCH --mem=56000M</td>
</tr>
<tr>
<td></td>
<td>#SBATCH --output=MPIBatchExample.%j</td>
</tr>
<tr>
<td></td>
<td>module load intel/2017b</td>
</tr>
<tr>
<td></td>
<td>mpirun prog.exe</td>
</tr>
<tr>
<td></td>
<td>bsub &lt; mpibatch.job</td>
</tr>
<tr>
<td></td>
<td>sbatch mpibatch.job</td>
</tr>
</tbody>
</table>
What is MPI

- **Message Passing Interface**: a specification for the library interface that implements message passing in parallel programming.

- Is standardized by the MPI forum for implementing portable, flexible, and reliable codes for distributed memory systems, regardless of the underneath architecture.
  - MPI-4.0 is under discussion

- Has C/C++/Fortran bindings.
  - C++ binding deprecated since MPI-2.2

- Different implementations (libraries) available: Intel MPI, MPICH, OpenMPI, etc.

- It is the most widely used parallel programming paradigm for large scale scientific computing.
Basic MPI Concepts

Communicator: MPI_COMM_WORLD
Size: 8
Rank: 0, 1, ..., 7

Communicator: comm1
Size: 2
Rank: 0, 1

Communicator: comm2
Size: 4
Rank: 0, 1, 2, 3

Communicator: comm3
Size: 3
Rank: 0, 1, 2

All MPI communication must specify a communicator.
Basic MPI Concepts

message, point-to-point communication, collective communication
Example 2 – One Sender and One Receiver

**C**

```c
#include <stdlib.h>
#include <mpi.h>

int main(int argc, char **argv){
    int np, rank, number;
    MPI_Status status;

    MPI_Init(&argc, &argv);
    MPI_Comm_size(MPI_COMM_WORLD, &np);
    MPI_Comm_rank(MPI_COMM_WORLD, &rank);

    if (rank == 0){
        number = 1234;
        MPI_Send(&number, 1, MPI_INT, 1, 0, MPI_COMM_WORLD);
        printf("Process %d sends out %d to process 1\n", rank, number);
    }else if(rank == 1){
        MPI_Recv(&number, 1, MPI_INT, 0, 0, MPI_COMM_WORLD, &status);
        printf("Process %d receives %d from process 0\n", rank, number);
    }
    MPI_Finalize();
}
```

**Fortran**

```fortran
program simple
use mpi
implicit none
integer ierr, np, rank, number, status ;

call MPI_INIT(ierr)

call MPI_COMM_SIZE(MPI_COMM_WORLD, np, ierr)
call MPI_COMM_RANK(MPI_COMM_WORLD, rank, ierr)

if (rank == 0) then
    number = 1234
    call MPI_SEND(number, 1, MPI_INTEGER 1, 0, MPI_COMM_WORLD, ierr)
    print *, "process ", rank, " sends ", number
else if (rank == 1) then
    call MPI_RECV(number, 1, MPI_INTEGER, 0, MPI_COMM_WORLD, status, ierr)
    print *, "process ",rank," receives ", number
endif

call MPI_Finalize(ierr)
end program simple
```
Communicator

- In MPI, a communicator is a software structure through which we specify a group of processes.
- Each process in a communicator is assigned a unique rank (an integer) ranging from 0 to (group_size -1). group_size is the size of the communicator.
- The constant MPI_COMM_WORLD (obtained from MPI include file) is an initial communicator that includes all the MPI processes activated in the running of a program. MPI_COMM_WORLD is typically the most used communicator.
- Communicators are especially useful in characterizing the tasks that different groups of processes carry out.
Size and Rank

- How many processes in a communicator?

<table>
<thead>
<tr>
<th>C</th>
<th>int MPI_Comm_size(MPI_Comm comm, int *size)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fortran</td>
<td>SUBROUTINE MPI_COMM_SIZE(comm, size, ierr)</td>
</tr>
<tr>
<td></td>
<td>integer comm, size, ierr</td>
</tr>
</tbody>
</table>

- What’s the rank (identity) of each process in a communicator?

<table>
<thead>
<tr>
<th>C</th>
<th>int MPI_Comm_rank(MPI_Comm comm, int *rank)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fortran</td>
<td>SUBROUTINE MPI_COMM_RANK(comm, rank, ierr)</td>
</tr>
<tr>
<td></td>
<td>integer comm, rank, ierr</td>
</tr>
</tbody>
</table>
Tag is an integer used in a message to differentiate one message from other messages.

Type of the data to be sent or received. Datatype can be predefined or user defined. Commonly used predefined datatypes, also called MPI basic datatypes in Fortran:

- MPI_INTEGER
- MPI_REAL
- MPI_REAL8
- MPI_CHARACTER
- MPI_LOGICAL
program simple
use mpi
implicit none
integer ierr, np, rank, number, status ;

call MPI_INIT(ierr)
call MPI_COMM_SIZE(MPI_COMM_WORLD, np, ierr)
call MPI_COMM_RANK(MPI_COMM_WORLD, rank, ierr)
if (rank == 0) then
    number = 1234
call MPI_SEND(number, 1, MPI_INTEGER, 1, 0, MPI_COMM_WORLD, ierr)
print *, "process ", rank, " sends ", number
else if (rank == 1) then
    call MPI_RECV(number, 1, MPI_INTEGER, 0, 0, MPI_COMM_WORLD, status, ierr)
print *, "process ", rank, " receives ", number
endif

call MPI_Finalize(ierr)
end program simpleC

Send and Receive a Message

MPI_SEND(number, 1, MPI_INTEGER, 1, 0, MPI_COMM_WORLD, ierr)
MPI_RECV(number, 1, MPI_INTEGER, 0, 0, MPI_COMM_WORLD, status, ierr)
Point-to-Point Communication

- Blocking: MPI_Send, MPI_Recv
- Non-blocking: MPI_Isend, MPI_Irecv
- Send-Receive: MPI_Sendrecv
## Blocking Send

### C

```c
int MPI_Send(void *buf, int count, MPI_Datatype datatype, int dest, int tag, MPI_Comm comm)
```

### Fortran

```fortran
MPI_SEND(buf, count, datatype, dest, tag, comm, ierr)
<type> buf(*)
integer count, datatype, dest, tag, comm, ierr
```

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>buf</td>
<td>initial address of send buffer</td>
</tr>
<tr>
<td>count</td>
<td>number of elements in send buffer</td>
</tr>
<tr>
<td>datatype</td>
<td>datatype of each send buffer element</td>
</tr>
<tr>
<td>dest</td>
<td>rank of destination</td>
</tr>
<tr>
<td>tag</td>
<td>message tag</td>
</tr>
<tr>
<td>comm</td>
<td>communicator</td>
</tr>
</tbody>
</table>
Comments on Blocking Send

MPI_SEND(buf, count, datatype, dest, tag, comm)

- The calling process causes count many contiguous elements of type datatype to be sent, starting from buf.
- The message sent by MPI_SEND can be received by either MPI_RECV or MPI_IRecv.
- MPI_SEND doesn’t return (i.e., blocked) until it is safe to write to the send buffer.
  - Safe means the message has been copied either into a system buffer, or into the receiver’s buffer, depending on which mode the send call is currently working under.
## Blocking Receive

<table>
<thead>
<tr>
<th>C</th>
<th>int MPI_Recv(void *buf, int count, MPI_Datatype datatype, int source, int tag, MPI_Comm comm, MPI_Status *status)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fortran</td>
<td>MPI_RECV(buf, count, datatype, source, tag, comm, status, ierr)</td>
</tr>
</tbody>
</table>

```c
<type> buf(*)
```

<table>
<thead>
<tr>
<th>Argument</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>buf</td>
<td>initial address of receive buffer</td>
</tr>
<tr>
<td>count</td>
<td>number of elements in receive buffer</td>
</tr>
<tr>
<td>datatype</td>
<td>datatype of each receive buffer element</td>
</tr>
<tr>
<td>source</td>
<td>rank of source or MPI_ANY_SOURCE</td>
</tr>
<tr>
<td>tag</td>
<td>message tag or MPI_ANY_TAG</td>
</tr>
<tr>
<td>comm</td>
<td>communicator</td>
</tr>
<tr>
<td>status</td>
<td>status object</td>
</tr>
</tbody>
</table>

*MPI_ANY_SOURCE and MPI_ANY_TAG are MPI defined wildcards.*
Comments on Blocking Receive

MPI_RECV(buf,count,datatype,source,tag,comm,status)

- The calling process attempts to receive a message with specified envelope (source, tag, communicator).
  - MPI_ANY_SOURCE and MPI_ANY_TAG are valid values.
- When the matching message arrives, elements of the specified datatype are placed in the buffer in contiguous locations, starting at the address of buf.
- The buffer starting at buf is assumed pre-allocated and has capacity for at least count many datatype elements.
  - An error returns if buf is smaller than data received.
Comments on Blocking Receive

MPI_RECV(buf, count, datatype, source, tag, comm, status)

- MPI_RECV can receive a message send by MPI_SEND or MPI_ISEND.
- Agreement in datatype between the send and receive is required.
- MPI_RECV is blocked until the message has been copied into buf.
- The actual size of the message received can be extracted with MPI_GET_COUNT.
Return Status

- The argument status in MPI_Recv provides a way of retrieving message source, message tag, and message error from the message.

- status is useful when MPI wildcards (MPI_ANY_SOURCE, MPI_ANY_TAG) are used in MPI_Recv.

- status can be ignored with MPI_STATUS_IGNORE

<table>
<thead>
<tr>
<th>C</th>
<th>Fortran</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPI_Status status</td>
<td>integer status(MPI_STATUS_SIZE)</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>MPI_Recv(...)</td>
<td>CALL MPI_RECV(...)</td>
</tr>
<tr>
<td>Source_id = status.MPI_SOURCE</td>
<td>source_id = status(MPI_SOURCE)</td>
</tr>
<tr>
<td>tag = status.MPI_TAG</td>
<td>tag = status(MPI_TAG)</td>
</tr>
</tbody>
</table>
#include <stdio.h>
#include <mpi.h>

int main(int argc, char **argv) {
    int number, size, rank;
    int i;

    MPI_Init(&argc, &argv);
    MPI_Comm_size(MPI_COMM_WORLD, &size);
    if (size < 2) {
        MPI_Abort(MPI_COMM_WORLD, 99);
    }
    MPI_Comm_rank(MPI_COMM_WORLD, &rank);
    if (rank == 0) {
        printf("Type any number from the input: ");
        scanf("%d", &number);
        for (i = 1; i < size; i++)
            MPI_Send(&number, 1, MPI_INT, i, 0, MPI_COMM_WORLD);
    } else {
        MPI_Recv(&number, 1, MPI_INT, 0, 0, MPI_COMM_WORLD,
                  MPI_STATUS_IGNORE);
        printf("My id is %d. I received %d\n", rank, number);
    }
    MPI_Finalize();
}

Example 3
• One sender and multiple receivers
• The sender sends out a number to each and every receiver

program ex3
use mpi
implicit none
integer rank, np, ierr, number, i

call MPI_INIT(ierr)
call MPI_COMM_SIZE(MPI_COMM_WORLD, np, ierr)
call MPI_COMM_RANK(MPI_COMM_WORLD, rank, ierr)

if (np < 2) then
call MPI_ABORT(MPI_COMM_WORLD, 99, ierr)
endif
if (rank == 0) then
    print *, "Type an integer from the input"
    read *, number
    do i = 1, np - 1
        call MPI_SEND(number, 1, MPI_INTEGER, i, 0, MPI_COMM_WORLD, ierr)
    enddo
else
    call MPI_RECV(number, 1, MPI_INTEGER, 0, 0, MPI_COMM_WORLD, &
                  MPI_STATUS_IGNORE, ierr)
    print "(2(A,I6))","Process ",rank, " received ", number
Endif
call MPI_FINALIZE(ierr)
end program ex3
Non-blocking Send

MPI_ISEND(buf, count, datatype, dest, tag, comm, request)

<table>
<thead>
<tr>
<th>Type</th>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IN</td>
<td>buf</td>
<td>initial address of send buffer</td>
</tr>
<tr>
<td>IN</td>
<td>count</td>
<td>number of elements in send buffer</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(non-negative integer)</td>
</tr>
<tr>
<td>IN</td>
<td>datatype</td>
<td>datatype of each send buffer element</td>
</tr>
<tr>
<td>IN</td>
<td>dest</td>
<td>rank of destination</td>
</tr>
<tr>
<td>IN</td>
<td>tag</td>
<td>message tag</td>
</tr>
<tr>
<td>IN</td>
<td>comm</td>
<td>communicator</td>
</tr>
<tr>
<td>OUT</td>
<td>request</td>
<td>communication request (a handle that can be used later to refer the outstanding receive)</td>
</tr>
</tbody>
</table>
Non-blocking Receive

MPI_Irecv(buf, count, datatype, source, tag, comm, request)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>buf</td>
<td>initial address of send buffer</td>
</tr>
<tr>
<td>count</td>
<td>number of elements in receive buffer (non-negative integer)</td>
</tr>
<tr>
<td>datatype</td>
<td>datatype of each receive buffer element</td>
</tr>
<tr>
<td>source</td>
<td>rank of source or MPI_ANY_SOURCE</td>
</tr>
<tr>
<td>tag</td>
<td>message tag or MPI_ANY_TAG</td>
</tr>
<tr>
<td>comm</td>
<td>communicator</td>
</tr>
<tr>
<td>request</td>
<td>communication request (a handle that can be used later to refer the outstanding receive)</td>
</tr>
</tbody>
</table>
Non-blocking Send/Receive

- A non-blocking send/receive call initiates the send/receive operation, and returns immediately with a request handle, before the message is copied out/into the send/receive buffer.

- A separate send/receive complete call is needed to complete the communication before the buffer can be accessed again.

- A non-blocking send can be matched by a blocking receive; a non-blocking receive can be matched by a blocking send.

- Used correctly, non-blocking send/receive can improve program performance.

- They also make the point-to-point transfers “safer” by not depending on the size of the system buffers.
  - No deadlock caused by unavailable buffer
  - No buffer overflow
Auxiliary Routines for Non-blocking Send/Receive

- Auxiliary routines are used to complete a non-blocking communication or communications.
- Commonly used auxiliary routines:

<table>
<thead>
<tr>
<th>Routine</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPI_Wait(request, status)</td>
<td>The calling process waits for the completion of a non-blocking send/receive identified by request.</td>
</tr>
<tr>
<td>MPI_Waitall(count, requests, statuses)</td>
<td>The calling process waits for all pending operations in a list of requests.</td>
</tr>
<tr>
<td>MPI_Test(request, flag, status)</td>
<td>The calling process tests a non-blocking send/receive specified by request has completed delivery/receipt of a message.</td>
</tr>
</tbody>
</table>
# MPI_WAIT

**MPI_WAIT(request, status)**

- **request**: request (handle)
- **status**: status object (Status)

## C
```
MPI_Request request;
MPI_Status status;
...
MPI_Irecv(recv_buf,count,...,
    comm,&request);
...do some computations ...
MPI_Wait(&request, &status);
```

## Fortran
```
integer request
integer status(MPI_STATUS_SIZE)
...
call MPI_Irecv(recv_buf,count,...&
    comm,request(ierr)
... do some computations ...
call MPI_WAIT(request,status(ierr)
```

*status* can be ignored with *MPI_STATUS_IGNORE*
MPI_WAITALL(count, requests, statuses)

C

```c
integer reqs[4]
integer statuses(MPI_STATUS_SIZE, 4)
...
call MPI_ISEND(..., reqs(1), ierr)
call MPI_IRecv(..., reqs(2), ierr)
call MPI_ISEND(..., reqs(3), ierr)
call MPI_IRecv(..., reqs(4), ierr)
...
... do some computations ...
... 
call MPI_WAITALL(4, reqs, statuses, ierr)
```

Fortran

```fortran
MPI_Request reqs[4];
MPI_Status status[4];
...
MPI_Isend(..., &reqs[0]);
MPI_Irecv(..., &reqs[1]);
MPI_Isend(..., &reqs[2]);
MPI_Irecv(..., &reqs[3]);
...
... do some computations ...
...
MPI_Waitall(4, reqs, statuses);
```

**statuses** can be ignored with MPI_STATUSES_IGNORE
## Example 4

<table>
<thead>
<tr>
<th>C</th>
<th>Fortran</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPI_Request *requests;</td>
<td>integer, allocatable::requests(:)</td>
</tr>
<tr>
<td>....</td>
<td></td>
</tr>
<tr>
<td>if (rank == 0){</td>
<td>if (rank == 0) then</td>
</tr>
<tr>
<td>printf(&quot;Type any number from the input: &quot;);</td>
<td>print *, &quot;Type an integer from the input&quot;</td>
</tr>
<tr>
<td>scanf(&quot;%d&quot;, &amp;number);</td>
<td>read *, number</td>
</tr>
<tr>
<td>requests = (MPI_Request <em>) (malloc(npof(MPI_Request)</em>(np-1)));</td>
<td>allocate(requests(np-1))</td>
</tr>
<tr>
<td>for (i=1; i&lt;np; i++)</td>
<td>do i=1, np-1</td>
</tr>
<tr>
<td>MPI_Isend(&amp;number, 1, MPI_INT, i, 0, MPI_COMM_WORLD,</td>
<td>call MPI_Isend(number, 1, MPI_INTEGER, i, 0, &amp;</td>
</tr>
<tr>
<td>&amp;requests[i-1]);</td>
<td>MPI_COMM_WORLD, ierr)</td>
</tr>
<tr>
<td>MPI_Waitall(np-1, requests, MPI_STATUSES_IGNORE);</td>
<td>enddo</td>
</tr>
<tr>
<td>free(requests);</td>
<td>call MPI_WAITALL(np-1, requests, MPI_STATUSES_IGNORE, ierr)</td>
</tr>
<tr>
<td>} else{</td>
<td>deallocate(requests)</td>
</tr>
<tr>
<td>MPI_Recv(&amp;number, 1, MPI_INT, 0, MPI_COMM_WORLD,</td>
<td>else</td>
</tr>
<tr>
<td>MPI_STATUS_IGNORE);</td>
<td>call MPI_RECV(number, 1, MPI_INTEGER, 0, 0, &amp;</td>
</tr>
<tr>
<td>printf(&quot;My id is %d. I received %d\n&quot;, rank, number);</td>
<td>MPI_COMM_WORLD, MPI_STATUS_IGNORE, ierr)</td>
</tr>
<tr>
<td>endif</td>
<td>print &quot;(2(A,I6))&quot;, &quot;Process &quot;,rank, &quot; received &quot;, number</td>
</tr>
<tr>
<td></td>
<td>endif</td>
</tr>
</tbody>
</table>
Send-Receive

MPI_SENDRECV(sendbuf, sendcount, sendtype, dest, sendtag, recvbuf, recvcount, recvtype, source, recvtag, comm, status)

- Combines send and receive operations in one call
- The source and destination can be the same.

- The message sent out by send-receive can be received by blocking/non-blocking receive or another send-receive
- It can receive a message sent by blocking/non-blocking send or another send-receive.
- Useful for executing a **shift operation** across a chain of processes.

- Dependencies will be taken care of by the communication subsystem to eliminate the possibility of deadlock.
A collective communication refers to a communication that involves all processes in a communicator.
# Routines for Collective Communication

<table>
<thead>
<tr>
<th>Routine</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPI_BARRIER</td>
<td>All processes within a communicator will be blocked until all processes within the communicator have entered the call.</td>
</tr>
<tr>
<td>MPI_BCAST</td>
<td>Broadcasts a message from one process to members in a communicator.</td>
</tr>
<tr>
<td>MPI_REDUCE</td>
<td>Performs a reduction operation to the vector of elements in the sendbuf of the group members and places the result in recvbuf on root.</td>
</tr>
<tr>
<td>MPI_GATHER</td>
<td>Collects data from the sendbuf of all processes in comm and place them consecutively to the recvbuf on root based on their process rank.</td>
</tr>
<tr>
<td>MPI_GATHERV</td>
<td>Collects data from the sendbuf of all processes in comm and place them consecutively to the recvbuf on root based on their process rank.</td>
</tr>
<tr>
<td>MPI_SCATTER</td>
<td>Distribute data in sendbuf on root to recvbuf on all processes in comm.</td>
</tr>
<tr>
<td>MPI_SCATTERV</td>
<td>Distribute data in sendbuf on root to recvbuf on all processes in comm.</td>
</tr>
<tr>
<td>MPI_ALLREDUCE</td>
<td>Same as MPI_REDUCE, except the result is placed in recvbuf on all members in a communicator.</td>
</tr>
<tr>
<td>MPI_ALLGATHER</td>
<td>Same as GATHER/GATHERV, except now data are placed in recvbuf on all processes in comm.</td>
</tr>
<tr>
<td>MPI_ALLGATHERV</td>
<td>Same as GATHER/GATHERV, except now data are placed in recvbuf on all processes in comm.</td>
</tr>
<tr>
<td>MPI_ALLTOALL</td>
<td>The j-th block of the sendbuf at process i is send to process j and placed in the i-th block of the recvbuf of process j.</td>
</tr>
</tbody>
</table>
MPI_BARRIER

MPI_BARRIER(comm)

- Blocks all processes in comm until all processes have called it.
- Is used to synchronize the progress of all processes in comm.
MPI_BCAST

MPI_BCAST(buffer, count, datatype, root, comm)

- Root process: sends a message to all processes (including root) in the communicator comm.
- Non-root processes: receives a message from the specified root.
- Each receiving process blocks until the message has arrived its buffer.
- All processes in comm must call this routine.
MPI_REDUCE

MPI_REDUCE(sendbuf, recvbuf, count, datatype, op, root, comm)

- Performs a reduction operation on all elements with same index in sendbuf on all processes and stores results in recvbuf of the root process.
- recvbuf is significant only at root.
- sendbuf and recvbuf cannot be the same.
- The size of sendbuf and recvbuf is equal to count.

C: MPI_Op op
Fortran: integer op

\[-\]

| 1.2345 | 2.4345 | -0.2374 |
| 3.1257 | 7.0321 | 3.2478  |
| 9.2134 | -0.9234| 0.2319  |

\[-\]

\[\begin{align*}
\text{MPI}_\text{SUM} & \quad \text{p0} \\
1.2345 & \quad 3.1257 & \quad 9.2134 \\
3.4306 & \quad 13.4056 & \quad 8.5219 \\
\end{align*}\]

\[-\]
## Predefined Reduction Operations

<table>
<thead>
<tr>
<th>PREDEFINED OPERATIONS</th>
<th>MPI DATATYPES</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPI_SUM, MPI_PROD</td>
<td>MPI_REAL8, MPI_INTEGER, MPI_COMPLEX,</td>
</tr>
<tr>
<td></td>
<td>MPI_DOUBLE, MPI_INT, MPI_SHORT, MPI_LONG</td>
</tr>
<tr>
<td>MPI_MIN, MPI_MAX</td>
<td>MPI_INTEGER, MPI_REAL8, MPI_INT, MPI_SHORT,</td>
</tr>
<tr>
<td></td>
<td>MPI_LONG, MPI_DOUBLE</td>
</tr>
<tr>
<td>MPI_LAND, MPI_LOR,</td>
<td>MPI_LOGICAL, MPI_INT, MPI_SHORT, MPI_LONG</td>
</tr>
<tr>
<td>MPI_LXOR</td>
<td></td>
</tr>
<tr>
<td>MPI_BAND, MPI_BOR,</td>
<td>MPI_INTEGER, MPI_INT, MPI_SHORT, MPI_LONG</td>
</tr>
<tr>
<td>MPI_BXOR</td>
<td></td>
</tr>
</tbody>
</table>
MPI_ALLREDUCE

MPI_ALLREDUCE(sendbuf, recvbuf, count, datatype, op, comm)

\[
\begin{align*}
\text{p0} & : 1.2345, 3.1257, 9.2134 \\
\text{p1} & : 2.4345, 7.0321, -0.9234 \\
\text{p2} & : -0.2374, 3.2478, 0.2319
\end{align*}
\]

\[
\begin{align*}
\text{p0} & : 3.4306, 13.4056, 8.5219 \\
\text{p1} & : 3.4306, 13.4056, 8.5219 \\
\text{p2} & : 3.4306, 13.4056, 8.5219
\end{align*}
\]

MPI_ALLREDUCE function is used to perform a reduction operation across all processes in a parallel computing environment.
**MPI_GATHER**

MPI_GATHER(sendbuf, sendcnt, sendtype, recvbuf, recvcnt, recvtype, root, comm)

- Gathers together data from all process in comm and stores in root process.
- Data received by root are stored in rank order.
- recvcnt is number of elements received per process.
-Recvbuf, recvcnt, recvtype are significant only at root.
MPI_GATHERV

MPI_GATHERV(sendbuf, sendcnt, sendtype, recvbuf, recvcnts, displs, recvtype, root, comm)

IN recvcnts  an integer array of size of comm. recvnts[i] = number of elements received from process i.
IN displs  an integer array of size of comm. displs[i] = displacement from recvbuf for process i.

Fortran
integer recvcnts(*), displs(*)

C
int recvcnts[], displs[]
MPI_GATHERV (cont.)

MPI_GATHERV(sendbuf,sendcnt,sendtype,recvbuf,recvcnts,displs, recvtype,root,comm)

C:
Fortran:

p0(root)
sendbuf

1.2345
3.1257

P1
sendbuf
2.4345
7.0321
-0.2374

P2
sendbuf
3.2478

recvbuf

recvcnts

displs
MPI_GATHERV (cont.)

MPI_GATHERV(sendbuf, sendcnt, sendtype, recvbuf, recvcnts, displs, recvtype, root, comm)
MPI_GATHERV (cont.)

MPI_GATHERV(sendbuf, sendcnt, sendtype, recvbuf, recvcnts, displs, recvtype, root, comm)

C:
Fortran:

<table>
<thead>
<tr>
<th>0</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>displs</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>recvnts</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>p0(root) sendbuf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2345 3.1257</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P1 sendbuf</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4345 7.0321 -0.2374</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P2 sendbuf</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2478</td>
</tr>
</tbody>
</table>

MPI_GATHERV (cont.)
MPI_ALLGATHER/MPI_ALLGATHERV

MPI_ALLGATHER(sendbuf, sendcnt, sendtype, recvbuf, recvcnt, recvtype, comm)

MPI_ALLGATHERV(sendbuf, sendcnt, sendtype, recvbuf, recvcnts, displs, recvtype, comm)

- Same as MPI_GATHER/MPI_GATHERV, except no root.
- Root is not needed since every process in the communicator stores the data gathered in its recvbuf.
MPI_SCATTER

MPI_SCATTER(sendbuf, sendcnt, sendtype, recvbuf, recvcnt, recvtype, root, comm)

```
p0
1.2345
3.1257

2.4345
7.0321

-0.2374
3.2478

p0
1.2345
3.1257

2.4345
7.0321

-0.2374
3.2478

p1

p2
```
MPI_SCATTERV

MPI_SCATTERV(sendbuf, sendcnts, displs, sendtype, recvbuf, recvcnt, recvtype, root, comm)

C:
Fortran:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>p0</th>
<th>p1</th>
<th>p2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2345</td>
<td>1.2345</td>
<td>1.2345</td>
<td></td>
</tr>
<tr>
<td>3.1257</td>
<td>3.1257</td>
<td>3.1257</td>
<td></td>
</tr>
<tr>
<td>2.4345</td>
<td>2.4345</td>
<td>2.4345</td>
<td></td>
</tr>
<tr>
<td>7.0321</td>
<td>7.0321</td>
<td>7.0321</td>
<td></td>
</tr>
<tr>
<td>-0.2374</td>
<td>-0.2374</td>
<td>-0.2374</td>
<td></td>
</tr>
<tr>
<td>3.2478</td>
<td>3.2478</td>
<td>3.2478</td>
<td></td>
</tr>
</tbody>
</table>

displs

| 2 | 3 | 1 |
recv cnts

<table>
<thead>
<tr>
<th>0</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>
Timing Routine

MPI_WTIME()

- returns a floating-point number in seconds, representing elapsed wall clock time since some time in the past.

```c
double t1, t2;
double elapsed;
t1 = MPI_Wtime();
...
// code segment to be timed
...
t2 = MPI_Wtime();
elapsed = t2 - t1;
```

```fortran
real*8 t1, t2
real*8 elapsed

! Code segment to be timed
...!
t1 = MPI_WTIME()
...
t2 = MPI_WTIME()
elapsed = t2 - t1
```
Example 5: Calculate PI

\[
\pi = \int_{0}^{1} \frac{4.0}{1 + x^2} \, dx
\]
Break
Outline for MPI Part II

- Exploring parallelism
  - Task-parallelism
  - Data-parallelism
- Matrix-vector multiplication
- Solving the 2D Poisson equation
- Domain decomposition
- MPI/OMP hybrid programming
- MPI/OMP: A Case Study
Important Note On Using MPI

- All parallelism is explicit: the programmer is responsible for correctly identifying parallelism and implementing parallel algorithms using MPI constructs.
Exploring Parallelism

- **Task-parallelism:** The programmer identifies different tasks of a program and distribute the tasks among different processors.

- **Data-parallelism:** The programmer partitions the data used in a program and distribute them among different processors, each performing similar operations on the subset of data assigned.
3 chefs need to prepare a three-course menu for 12 guests

Preparing 12 salads
Preparing 12 steaks
Preparing 12 deserts

Task parallelism

Task 1
Task 2
Task 3

Task 1:
Salad
Steak
Desert

Task 2:
Salad
Steak
Desert

Task 3:
Salad
Steak
Desert

Data parallelism

4 meals
Salad
Steak
Desert

4 meals
Salad
Steak
Desert

4 meals
Salad
Steak
Desert
Example 6: Data Distribution

**Cyclic Distribution**

Data is distributed in a round robin manner among the processes.

<table>
<thead>
<tr>
<th>Process id</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data id</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

**Fortran**

```fortran
do i=myid+1, N, nprocs
  x = h*(i-0.5d0)
  sum = sum+4.0d0/(1.0d0+x*x)  
enddo
sum = sum*h
```

**C**

```c
for (i=myid+1; i<=N; i+=nprocs){
  x = h*(i-0.5);
  sum += 4.0/(1.0+x*x);
}
sum = sum*h;
```

calc_PI_cyclic.c
Example 6: Data Distribution

Data is partitioned into $n$ contiguous parts, where $n$ is equal to the number of processes. Each process will take one part of the data.

<table>
<thead>
<tr>
<th>Process id</th>
<th>data id</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>

**C**

```c
block_map(1,N,nprocs,myid,&l1,&l2);
for (i=l1; i<=l2; i++){
    x = h*(i-0.5);
    sum += 4.0/(1.0+x*x);
}
sum = sum*h;
```

**Fortran**

```fortran
call block_map(1,N,nprocs,myid,l1,l2)
do i=l1, l2
   x = h*(i-0.5d0)
   sum = sum + 4.0d0/(1.0d0+x*x)
endo
sum = sum*h
```
Example 6: Data Distribution

Block Distribution

Process id

<table>
<thead>
<tr>
<th>data id</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block Distribution</td>
<td></td>
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<td></td>
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<tr>
<td>Process id</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

C

```c
void block_map(int n1, int n2, int nprocs, int myid, int *l1, int *l2) {
    int block, rem;
    block = (n2-n1+1)/nprocs;
    rem = (n2-n1+1)%nprocs;
    if (myid < rem) {
        block++;
        *l1 = n1+myid*block;
    } else {
        *l1 = n1+rem+block*myid;
    }
    *l2 = *l1+block-1;
}
```

Fortran

```fortran
subroutine block_map(n1,n2,nprocs,myid,l1,l2)

Integer n1, n2, nprocs, myid, l1, l2

integer block, rem
block = (n2-n1+1)/nprocs
rem = mod(n2-n1+1, nprocs)
if (myid < rem) then
    block = block+1
    l1 = n1+myid*block
else
    l1 = n1+rem+block*myid
end if
l2 = l1+block-1
end subroutine block_map
```
Example 6: Data Distribution

Data is divided into chunks of contiguous blocks and the chunks are distributed in a round-robin manner.

<table>
<thead>
<tr>
<th>Block-Cyclic Distribution</th>
<th>Process id</th>
<th>data id</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>2</td>
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<td>1</td>
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<tr>
<td></td>
<td>0</td>
<td>7</td>
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<tr>
<td></td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

### C

```c
for (i=myid*BLK+1; i<=N; i+=nprocs*BLK){
    for (j=i; j<=MIN(N,i+BLK-1); j++){
        x = h*(j-0.5);
        sum += 4.0/(1.0+x*x);
    }
}
sum = sum*h;
```

### Fortran

```fortran
do i=myid*BLK+1, N, nprocs*BLK
    do j=i, MIN(N,i+BLK-1)
        x = h*(j-0.5d0)
        sum = sum+4.0d0/(1.0d0+x*x)
    enddo
endo do
sum = sum*h
```

calc_PI_bc.c
Example 7: matvec-scatterv

\[
A \tilde{b} = (\tilde{a}_1 \ldots \tilde{a}_n) \tilde{b} = b_1 \tilde{a}_1 + b_2 \tilde{a}_2 + \cdots + b_n \tilde{a}_n = \tilde{c}
\]

\(\tilde{a}_i\) is a column vector.

In this program, strips of consecutive columns of \(A\) are distributed to all processes. Each process carries out a part of the linear vector sum

\[
b_i \tilde{a}_i + \cdots + b_j \tilde{a}_j
\]
Example 8: Solving the x-y Poisson Equation

Solve the partial differential equation

\[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y) \]

where \( x, y \in [0,1] \)

and \( u = g(x, y) \) on boundary

using the finite difference method.

Discretize the domain along \( x \) and \( y \) using \( n \) internal points in each direction.

The increment is \( h = 1/(n + 1) \)

\[ x_i = ih, \quad y_j = jh \]

\[ 0 \leq i, j \leq n + 1 \]

\[ u_{ij} = u(x_i, y_j) = u(ih, jh) \]

\[ 0 < i, j < n + 1 \]
Example 8: the x-y Poisson Equation

5-point finite difference stencil approximation:

\[ u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} - 4u_{i,j} = h^2 f_{i,j} \quad (0 < i, j < n + 1) \]

\[ u_{i,j} = 0.25 \times (u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} - h^2 f_{i,j}) \]

k+1 Jacobi iteration step at \( x_i = ih; \ y_j = jh; i, j = 1: n \)

\[ u^{k+1}_{i,j} = \frac{1}{4}(u^k_{i-1,j} + u^k_{i,j+1} + u^k_{i,j-1} + u^k_{i+1,j}) - h^2 f_{i,j} \]

Jacobi iteration across all points:

\[ \text{do } j = 1, n \]
\[ \quad \text{do } i = 1, n \]
\[ \quad \quad \text{unew}(i, j) = 0.25 \times (u(i-1,j) + u(i, j+1) + u(i+1,j) + i(i, j-1)) - f(i,j) \times h^2 \]
\[ \quad \text{end do} \]
\[ \text{end do} \]
Example 8: Solving the x-y Poisson Equation

Boundary conditions (stay fixed):

\begin{align*}
  u(x_i, 0) &= \frac{\cos(\pi x_i) - \pi^2}{\pi^2} \quad (0 \leq x \leq 1) \\
  u(x_i, 1) &= \frac{\cos(\pi x_i) - \pi^2 \cos(x_i)}{\pi^2} \quad (0 \leq x \leq 1) \\
  u(0, y_j) &= \frac{1}{\pi^2} - 1 \quad (0 \leq y \leq 1) \\
  u(1, y_j) &= -\left(\frac{1}{\pi^2} + \cos(y_j)\right) \quad (0 \leq y \leq 1)
\end{align*}

RHS: \( f(x_i, y_j) = (x_i^2 + y_j^2)\cos(x_iy_j) - \cos(\pi x_i) \)
1d-Domain Decomposition

C: decompose along y axis

Fortran: decompose along x axis
1d-Domain Decomposition

Border columns need to be copied into the memory space of the neighboring processes for the five-point stencil calculation. Same for row-wise decomposition.
1d-Domain Decomposition

Exchanged border columns are stored in ‘ghost’ columns in each process to be used in five-point stencil calculation.
**MPI/OpenMP Hybrid Programming**

- **Simplest and intuitive form:** master-only: only master thread can execute MPI calls

  ```
  Call MPI_INIT(ierr) 
  ...
 .Call MPI_SEND(...)
  ...
  !$OMP DO
do i=1, N
  ...
  ENDDO
  !$OMP END DO
  ...
  CALL MPI_FINALIZE(ierr)
  ```

- **Starting MPI-2, the standard provides guidelines on how to interact MPI with threads**

- **Four levels of thread support**
  - MPI_THREAD_SINGLE: Only one thread will execute
  - MPI_THREAD_FUNNELED: Only master thread will make MPI-calls
  - MPI_THREAD_SERIALIZED: Multiple threads may make MPI-calls, but only one at a time
  - MPI_THREAD_MULTIPLE: Multiple threads may call MPI, with no restrictions

  ```
  mpiifort -qopenmp [options] prog.f90 -o prog.exe
  ```
Example 9: Hybrid Programming

Int MPI_Init_thread(int *argc, char **argv, int required, int *provided)

MPI_INIT_THREAD(required, provided, ierr)

- Four possible values for the parameter `required`:
  - MPI_THREAD_SINGLE
  - MPI_THREAD_FUNNELED
  - MPI_THREAD_SERIALIZED
  - MPI_THREAD_MULTIPLE

```
int MPI_Init_thread(int *argc, char **argv, int required, int *provided) {

  MPI_INIT_THREAD(required, provided, ierr)
}
```
### Example 9: poisson_1d_hybrid

- **Master-only implementation**

- **Performance comparison using a 800x800 mesh**

<table>
<thead>
<tr>
<th>Pure MPI Single-node</th>
<th>Pure MPI 2-node</th>
<th>Hybrid OMP_NUM_THREADS=2</th>
<th>Hybrid OMP_NUM_THREADS=4</th>
</tr>
</thead>
<tbody>
<tr>
<td>np=8 80.37s</td>
<td>np=8,ppn=4 84.14s</td>
<td>np=4 161.18s</td>
<td>np=2 312.33s</td>
</tr>
<tr>
<td>np=16 43.24s</td>
<td>np=16,ppn=8 46.04s</td>
<td>np=8 84.57s</td>
<td>np=4 162.67s</td>
</tr>
<tr>
<td>np=20 36.48s</td>
<td>np=20,ppn=10 39.19s</td>
<td>np=10 72.21s</td>
<td>np=5 130.51s</td>
</tr>
<tr>
<td>np=40,ppn=20 25.88s</td>
<td>np=20,ppn=10 52.37s</td>
<td>np=10,ppn=5 74.02s</td>
<td></td>
</tr>
</tbody>
</table>

- **Not all program can benefit from hybrid programming**
Acknowledgement

- Part of the content is adapted from former Supercomputing Facility staff Mr. Spiros Vellas’ introductory MPI short course