Using computers to go where fluid dynamics experiments cannot

Fluids, Turbulence and Fundamental Transport Lab
Mechanical Engineering, Texas A&M

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• Heat transfer analysis in internal turbine cooling

• Passive scalar separation using chaotic advection
Figure: http://www.milnet.com/jeteng.htm
Motivation - From Flying and Thermo

Efficiency as a function of temperature ratio: 

$$\eta_{cycle} = 1 - \frac{T_5}{T_4}$$

Increase $T_4$, Limits: Metal melting temperature and part life
Transient hot spots can cause part failure

Figure: Turbine blade leading edge region, Right: from Langston (1980)
Figure: PDF measurements from Radomsky et al. (2000)
A highly resolved LES simulation is proposed

The large scales are solved on the grid while subgrid scales are modelled.

\[ \nabla \cdot \mathbf{U} = 0, \quad (1) \]
\[ \partial_t \mathbf{U} + \mathbf{U} \cdot \nabla \mathbf{U} = -\rho^{-1} \nabla P + \nabla \cdot ([\nu + \nu_t] \nabla \mathbf{U}), \quad (2) \]
\[ \partial_t T + \mathbf{U} \cdot \nabla T = \nabla \cdot ([\alpha + \alpha_t] \nabla T), \quad (3) \]

- Initial estimates based on a steady RANS computation (Knost et al. 2009) at \( \text{Re}_{\text{Chord}} \approx 150,000 \):
  - \( 10^8 \) cells (for \( x^+ \approx y^+ \approx z^+ \approx 50 \))
  - \( 10^6 \) time steps per flow through (based on CFL)
- Highly scalable, open-source Spectral Element code

2D basis function, \( N=10 \)

Fischer et al. 2007

- Strong scaling for 7.8 mio grid points
• 2D-periodic, divergence free solution of Navier-Stokes (Taylor vortices)
- Increased grid density near wall
- Length scale by grid spacing
- Freestream intensity by inflow vortex strength
- Boundary layer by slope and length of converging section
Preliminary results
Preliminary results
Preliminary results
Efficient Mixing in Laminar Flows Through Chaotic Particle Trajectories

- Exponential stretching of interface across which diffusion occurs
- Can be generated from simple flow fields.

Source: Scientific American, January 1989

Braiding with "Ghost-Rods"

Stirring in a braiding motion with physical rods

Physical rods replaced by periodic orbits

\[ u = \frac{\partial \psi}{\partial y} = \pm \sum_{n=1}^{N} U_n \sin(nx/2) \]

\[ \psi = 0 \]

\[ u = \frac{\partial \psi}{\partial y} = \mp \sum_{n=1}^{N} U_n \sin(nx/2) \]

"Stirring with ghost rods in a lid-driven cavity," by Pankaj Kumar, Jie Chen, and Mark Stremler.

The Chebychev-Fourier Method was used to solve a Vorticity-Stream Function formulation

source: Roger Peyret, Spectral Methods for Incompressible Viscous Flow, 2002

Convergence Plot for Stream Function, \( \text{Re} = \frac{U_{\text{max}} x h}{\nu} \)

Contours of Stream Function
Contour Plots of Stream Function

(a) \( \text{Re} = 0.1 \)

(b) \( \text{Re} = 1 \)

(c) \( \text{Re} = 10 \)

(d) \( \text{Re} = 100 \)
Mixing Index for Passive Scalar Transport

A. Duggleby  M. Schwänen  P. Rao  May 6, 2010  18/25
Mixing Index for Passive Scalar Transport

Mixing Index: 
\[ M = \frac{1}{N} \sum_{i=1}^{N} \frac{\theta_0 - |\theta_i - \theta_0|}{\theta_0} \]

ReSc = 10,000

Re = 0.1, t = 1.00[s]

Re = 1, t = 1.00[s]

Re = 10, t = 1.00[s]

Re = 100, t = 1.00[s]
Dispersion of Particles

Re = 0.1, time = 0.00[s]
Stirring Index for Different Re Based on the Box Counting Method

Stirring Index 1

Stirring Index 2

(a) Stirring Index 1

(b) Stirring Index 2

Stirring Index: \( \epsilon = \frac{1}{K} \sum_{i=1}^{K} \omega_i \)

\( \omega_i = \begin{cases} 
\frac{n_i}{n_{\max}}, & n_i < n_{\max} \\
1, & n_i \geq n_{\max} 
\end{cases} \)
Comparison of Dispersion of Particles between Re = 0.1 and Re = 10

(a) Re = 0.1, 8 Advection Cycles
(b) Re = 10, 8 Advection Cycles
(c) Re = 0.1, 18 Advection Cycles
(d) Re = 10, 18 Advection Cycles
Separation of Substances with Close Diffusivities

(Re=0.01, Sc=\(\frac{\nu}{D}\) = 10\(^6\), \(\chi = \frac{D_1 - D_2}{D} = 0.05\))

(a) Contours of \(\theta_1\)

(b) Contours of \(\theta_2\)

(c) Contours of \(\theta_1\) after unbraiding

(d) Contours of \(\theta_1 - \theta_2\)
Chaotic Separation vs. diffusion

\[ A^{-1} \int |\theta_1 - \theta_2| \, dA \]

- **Plain Diffusion**
- 1 cycle
- 2 cycles
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Texas A&M Supercomputing Center has played an important role in this work.