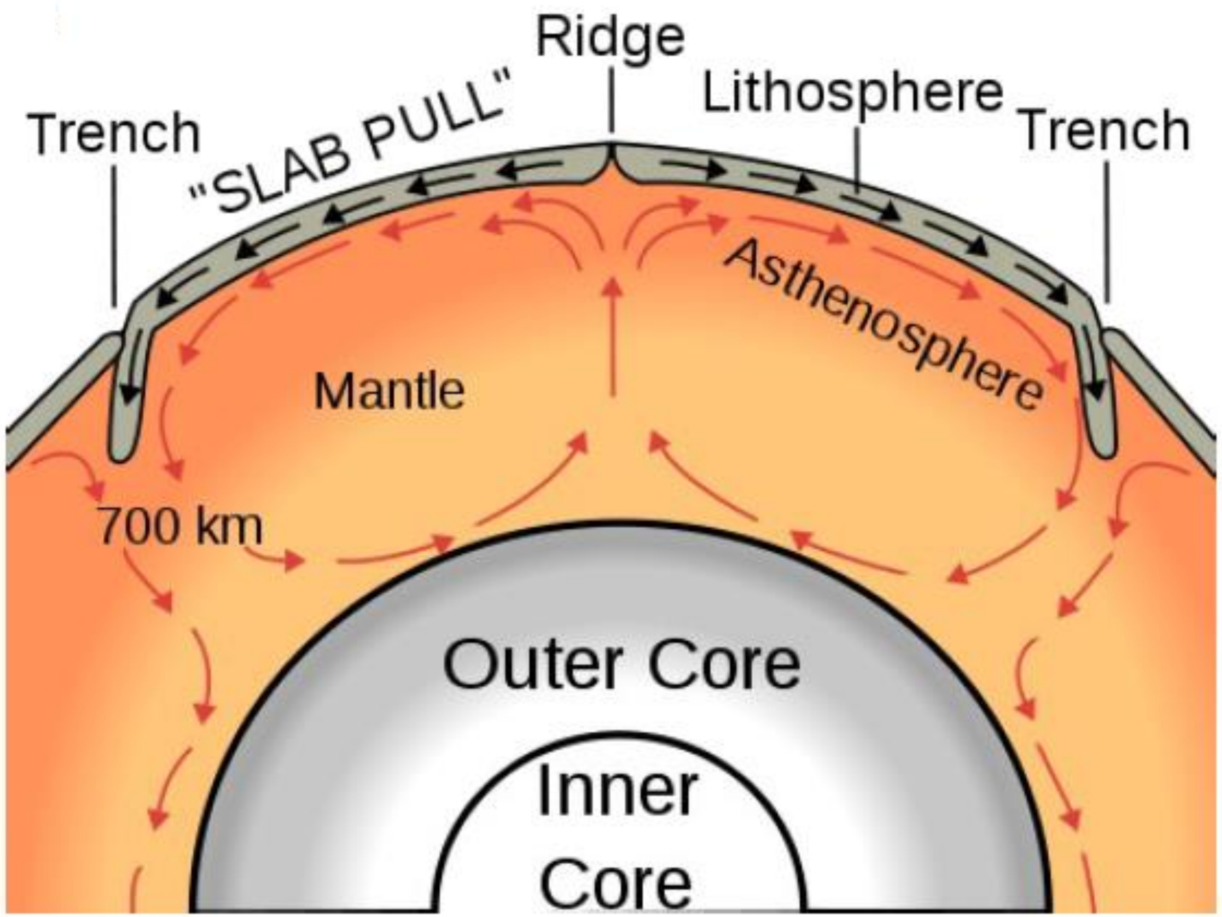
Wolfgang Bangerth, Department of Atmospheric Sciences, Texas A&M University
Joint work with Timo Heister, Eric Heien, Thomas Geenen, Martin Kronbichler, Juliane Dannberg, Rene Gassmoeller and many many other contributors

The what and the why

Brief recap on the Earth mantle:

- It makes up the largest component of Earth (~80%)
- It is solid
- It flows on long time scales
- Thermal gradients drive convection

- It is probably Earth's component we understand the least
- Yet, it has a large impact on basically everything else
- It's relevance lies in the interaction with the rest of Earth



The what and the why

“Big” questions one may ask about the mantle in relation to other systems:

- *Lithosphere:* How does mantle convection interact with plate tectonics?
- *Atmosphere:* Participation in the carbon cycle?
  Oceans: Participation in the water cycle?
  - implications on habitability of planets
- *Core:* Heat transport from core to surface?
  - impact on the magnetic field
  - thermal history of Earth
  - history of the inner core

The what and the why

The only way to answer all of these questions:

Computer Simulation

Our tool:

ASPECT – the Advanced Simulator for Problems in Earth ConvecTion.

ASPECT is open source; see *http://aspect.dealii.org/*

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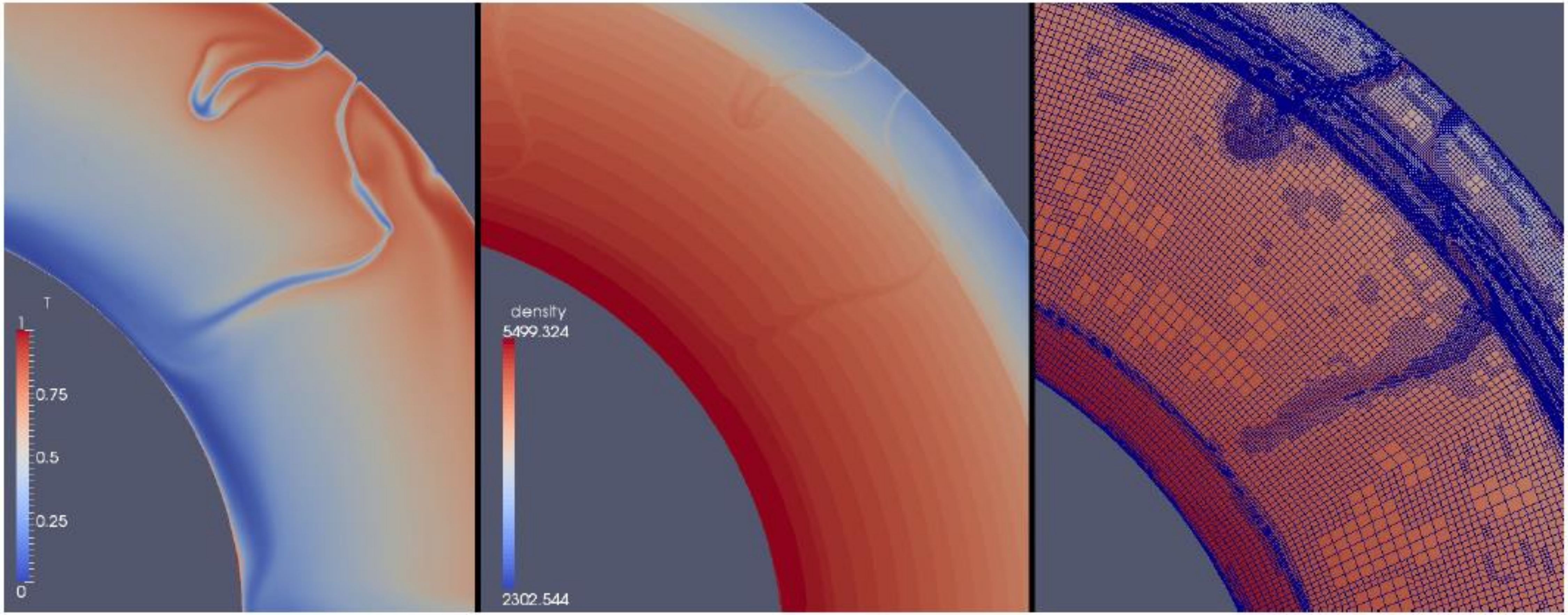
Examples

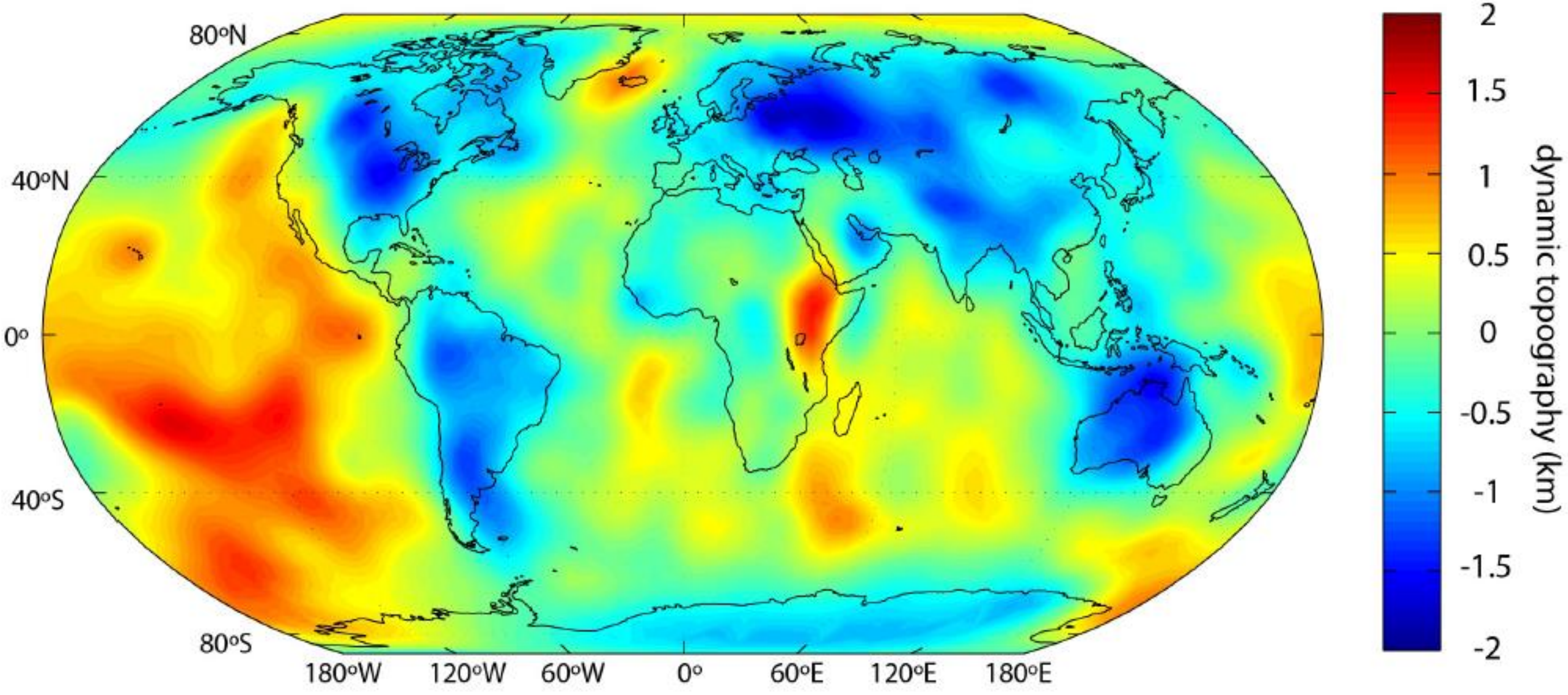
Thomas Geenen (Utrecht, The Netherlands):

Phase changes and their influence on subduction.



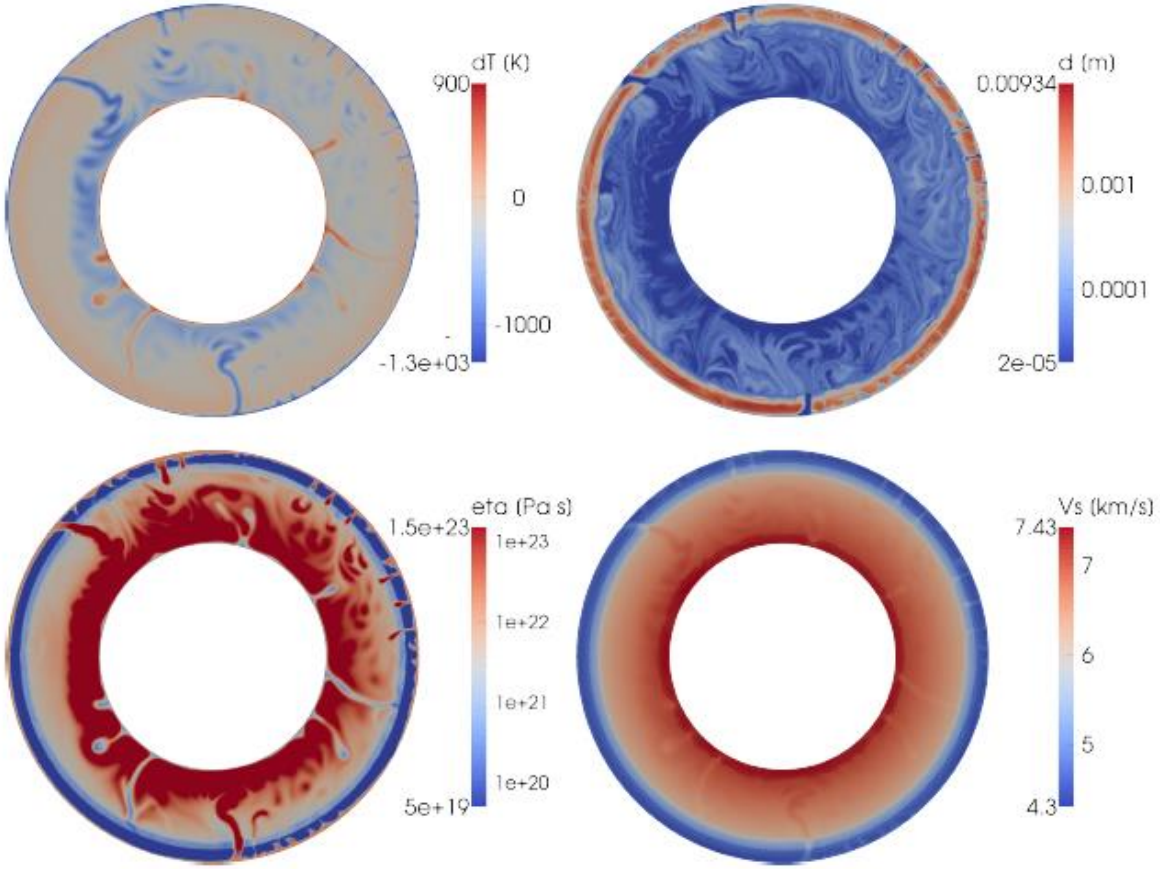
Examples

Jacky Austermann, Jerry Mitrovica (Harvard):
Determine the role of mantle convection in the dynamic (paleo-)topography (free surface) of the Earth.



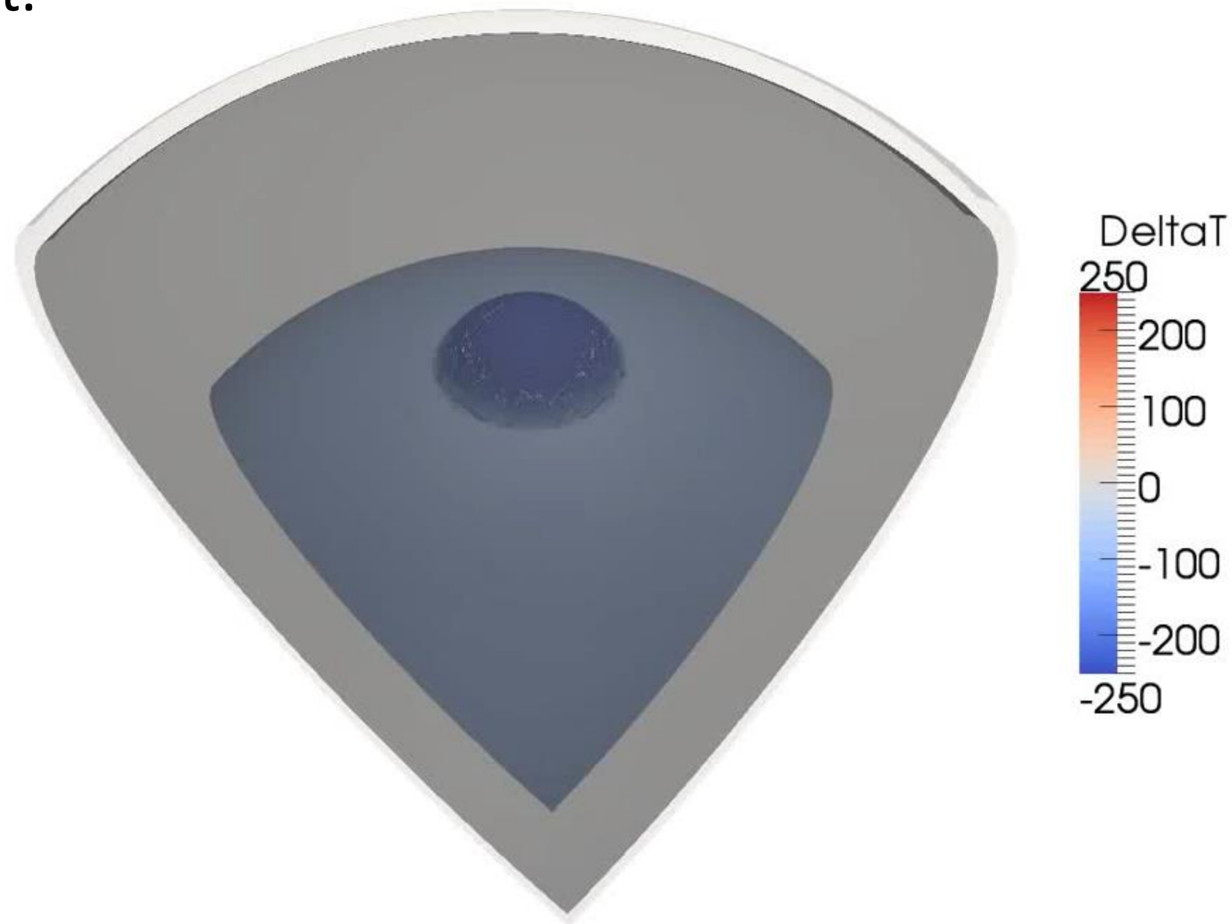
Examples

Juliane Dannberg et al. (GFZ Potsdam, now at TAMU):
Grain size evolution and its influence on seismic velocities.



Examples

Juliane Dannberg et al. (GFZ Potsdam, now at TAMU):
Migration of melt.



Design goals

Aspect – the *Advanced Solver for Problems in Earth's ConvecTion* – is a "community code":

- Can solve problems of interest (to geodynamicists)
- Is well tested
- Uses modern numerical methods

- Is very well documented
- Designed to be easy to extend

- Presents interesting mathematical problems worth exploring

Challenges: Problem size

For (global) convection in the earth mantle:

• Depth:	~35 – 2890 km
• Volume:	~10^{12} km^3
• Resolution required:	<10 km
• Uniform mesh:	~10^9 cells
• Using Taylor-Hood (Q2/Q1) elements:	33B unknowns

- At 100k-1M DoFs/processor: 30k-300k processors!

Consequence: We need parallelism, adaptive mesh refinement.

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Challenges: Model complexity

Thermal convection is described by the relatively "simple" Boussinesq approximation:

$$-\nabla\cdot[2\eta\epsilon(u)]+\nabla p=g\rho(T)$$

$$\nabla\cdot u=0$$

$$\frac{\partial T}{\partial t}+u\cdot\nabla T-\kappa\Delta T=\gamma+\alpha\left(\frac{\partial p}{\partial t}+u\cdot\nabla p\right)+\eta(\nabla u)^2$$

Problem: Every coefficient here is strongly nonlinear.

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Solutions

Among the mathematical techniques we use are:

- Higher order time stepping schemes
- Higher order finite elements
- Fully adaptive, dynamically changing 3d meshes
- Iterate out the nonlinearity
- Silvester/Wathen-style block preconditioners with F-GMRES
- Algebraic multigrid for the elliptic part
- Parallelization using MPI, threads, and tasks

To make the code usable by the community:

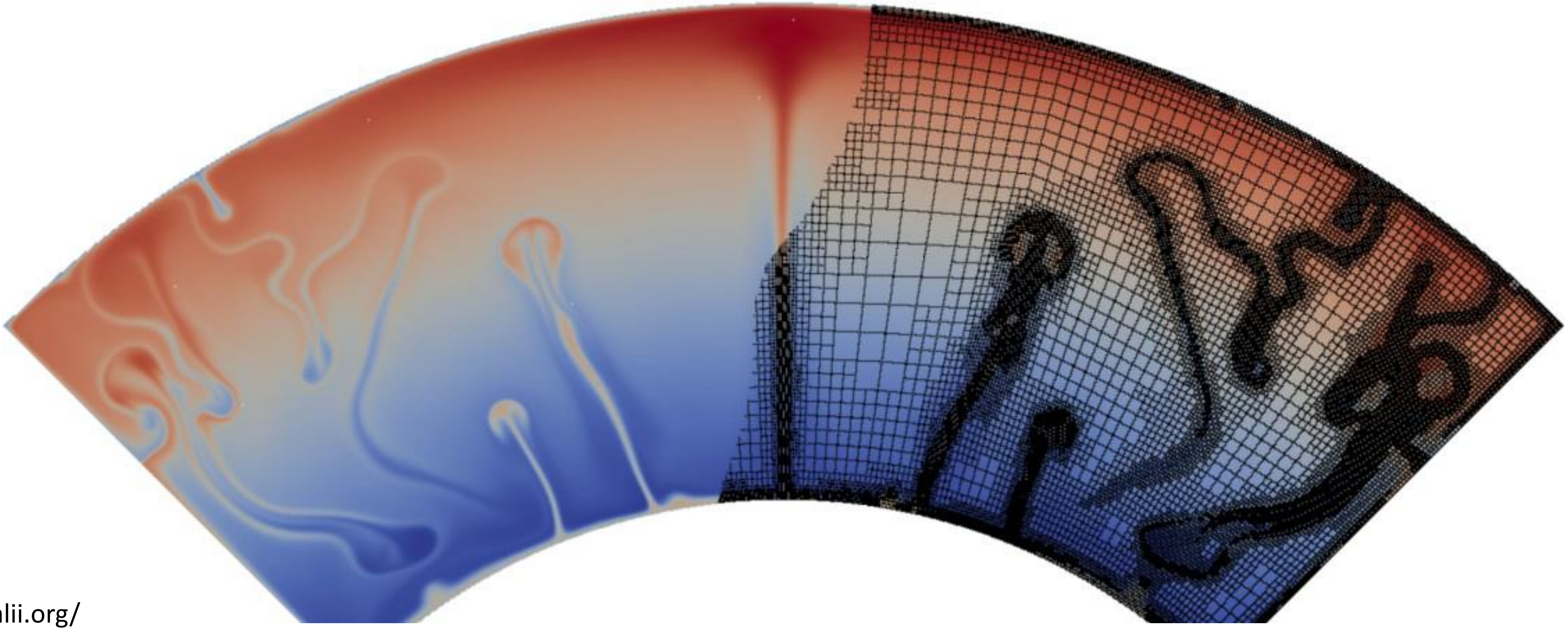
- Use object-oriented programming, build on external tools
- Make it modular, separate concerns
- Extensive documentation
- Extensive and frequent testing

Choose the most efficient techniques for *every* piece of the puzzle!

Features of ASPECT: Adaptivity

Adaptivity:

- The mesh does not have to be fine everywhere
- Automatically refine and coarsen it where and when necessary



http://www.dealii.org/

Features of ASPECT: Stokes solvers

Solvers:

- The most efficient kinds of solvers today are of Krylov type (CG, GMRES, MinRes, …)
- However, they need good preconditioners

- Here, we want to solve a Stokes system $\begin{pmatrix} A & B \\ B^T & 0 \end{pmatrix} \begin{pmatrix} U \\ P \end{pmatrix} = \begin{pmatrix} F \\ 0 \end{pmatrix}$

- The best preconditioners have the form $\begin{pmatrix} A & B \\ 0 & S \end{pmatrix}^{-1}$

with $S = B^T A^{-1} B$

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Features of ASPECT: Stokes solvers

Preconditioner: In isoviscous case, Silvester-Wathen preconditioner

$$\begin{pmatrix} A & B \\ 0 & S \end{pmatrix}^{-1} \approx \begin{pmatrix} \widetilde{A^{-1}} & B \\ 0 & M^{-1} \end{pmatrix} \qquad \text{or} \quad \approx \begin{pmatrix} A^{-1} & B \\ 0 & M^{-1} \end{pmatrix}$$

works very well:

- 30-50 GMRES iterations
- 8-10 inner iterations for *A*

Problem: For non-isoviscous problems:

- Some cases take 100s of GMRES iterations
- Some cases take 100s of inner iterations for *A*

Solution: Averaging material properties helps!

Features of ASPECT: Parallelization

Parallelization: Strong scaling on Cray XC-40 (Stuttgart, Germany)

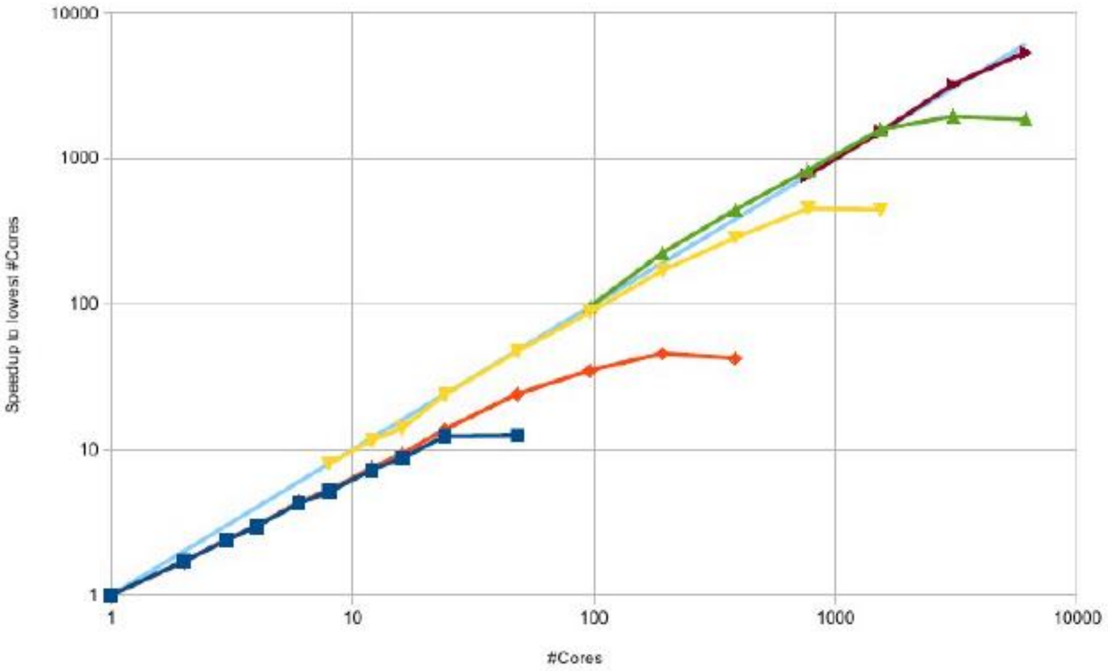


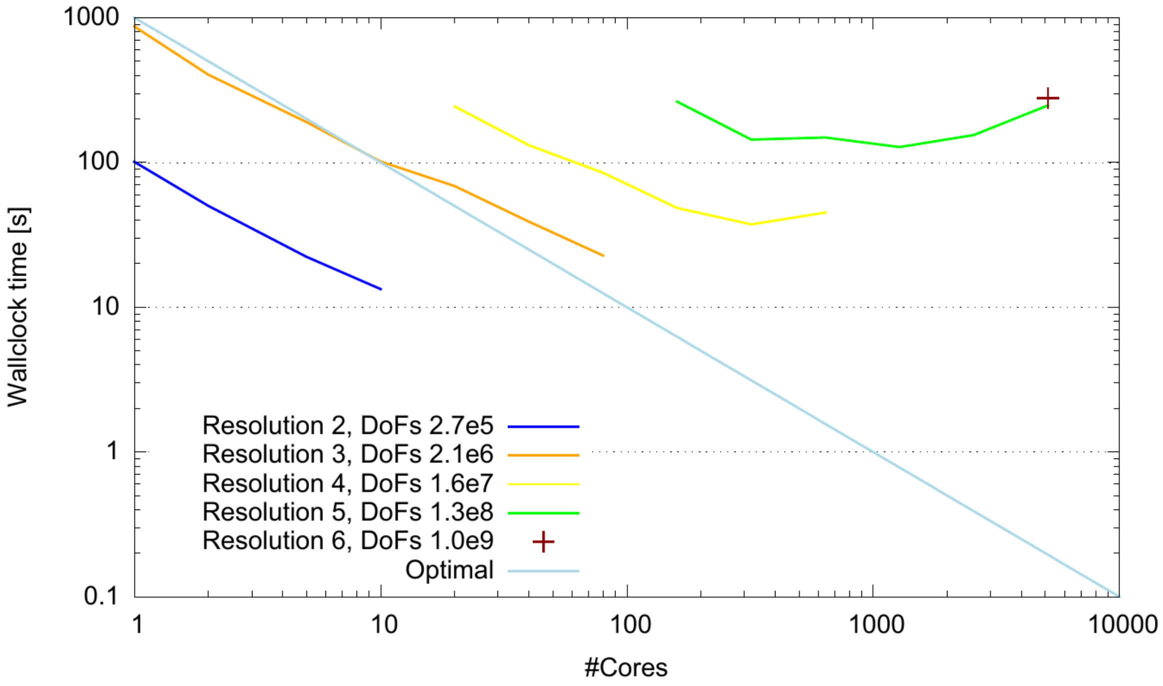
Fig.3: Scalability results for the ASPECT code on up to 8000k cores and more than 300M unknowns.
(Credit: R. Gassmoeller)

As long as we have >50,000 DoFs per processor, we get

- almost linear strong scaling of CPU time
- linear weak scaling of CPU time

Features of ASPECT: Parallelization

Parallelization: Preliminary data on Texas A&M's *ada* cluster



Credit: R. Gassmoeller, F. Dang

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Features of ASPECT: Parallelization

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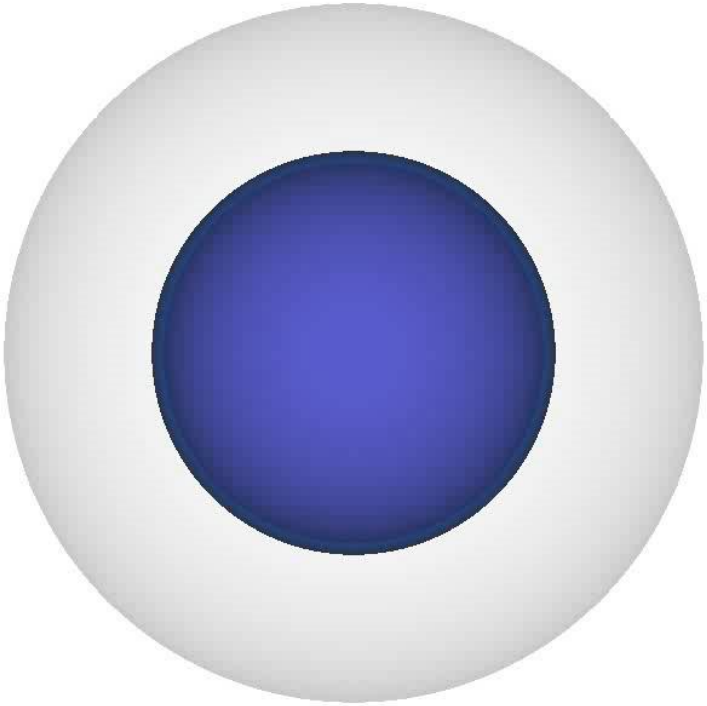
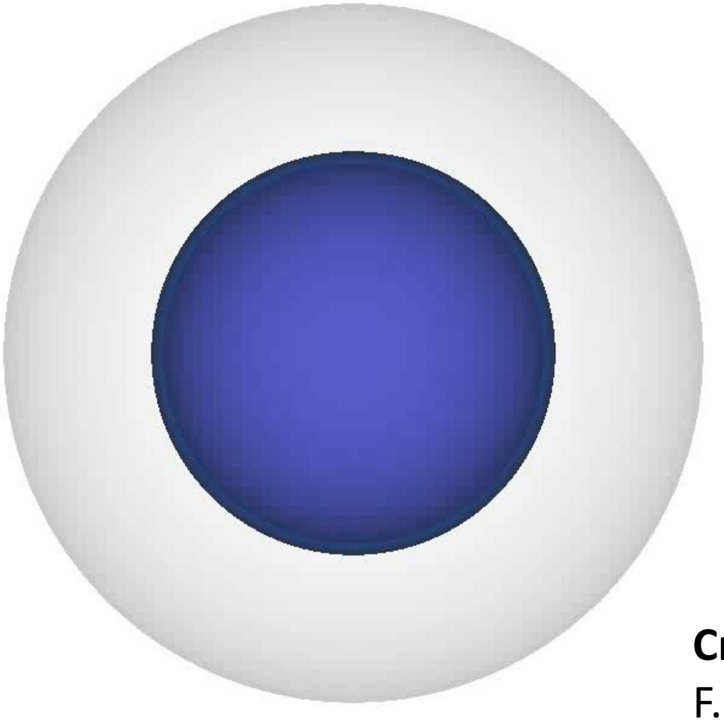




Credit: R. Gassmoeller, F. Dang

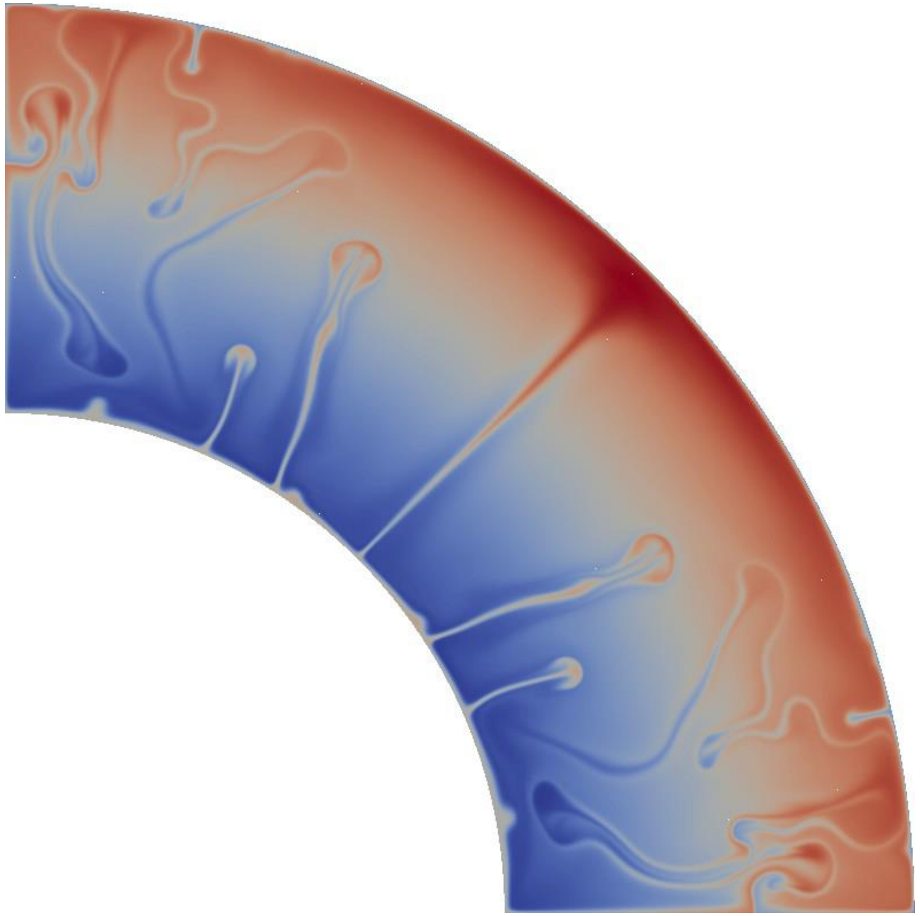
100 million degrees of freedom, 4500 model timesteps, 5 days on 2000 cores

Conclusions

We can only understand mantle convection through computer simulation

***Aspect* is a code written for this:**

- Uses modern numerical methods
- Accurate
- Fast, scalable
- Well tested, well documented
- Designed to be easily extended



Conclusions

Aspect – Advanced Solver for Problems in Earth's ConvecTion:

http://aspect.dealii.org/

Reference:

M. Kronbichler, T. Heister, W. Bangerth:
High accuracy mantle convection simulation through modern numerical methods.
Geophysics Journal International, 2012.