Introduction: MH370

On March 8, 2014 Malaysia Airlines Flight MH370 disappeared less than an hour after take-off on a flight from Kuala Lumpur to Beijing. The Boeing 777-200ER carried 12 crew members and 227 passengers.

“It is therefore with deep sadness and regret that I must inform you that ... Flight MH370 ended in the Southern Indian Ocean.”
— Malaysia Prime Minister Perdana Menteri
Introduction: An Interdisciplinary Perspective

Research on this air incident requires interdisciplinary perspective. Computational mathematics and mechanics can help us:

- understand the physical nature of an aircraft emergency water landing
- model and compute this problem
- use this knowledge to help safe civil aviation and other aerospace related undertakings

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Challenges for Classical Methods

Sub-models and corrections are needed for various complicating factors

- trapped air cavitation
- water compressibility and acoustic effects
- complex, real-world geometries
- ...

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Approach and Objective

Flight MH370:
• If flight MH370 did not have a mid-air explosion, then all available signs indicate that it crashed somewhere in the Indian Ocean. This is an aircraft water-entry problem.

Approach:
• Instead of the classical methods introduced above, we utilize Computation Fluid Dynamics (CFD) that can resolve local details and take factors like trapped air, water compressibility, and the real-world geometry (Boeing 777) into account.

Objective:
• Numerically simulate and analyze several hypothetical scenarios.

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OpenFOAM: Software Platform

OpenFOAM (Open Source Field Operation and Manipulation) is an open-source C++ toolbox for the development of customized numerical solvers, and pre-/post-processing utilities for the solution of continuum mechanics problems, including CFD.

There are three steps to run OpenFOAM:

• generate polyhedral mesh
• execute a numerical solver for the given differential equation
• display and analyze the results
Aircraft Water Entry

θ = pitch angle
β = angle of approach

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Governing Equations I

- **(Conservation of phase mass)**
  \[
  \frac{\partial (\rho_i \alpha_i)}{\partial t} + \nabla \cdot (\rho_i \alpha_i \mathbf{u}) = 0.
  \]
  \(\alpha_i\) = volume fraction of phase \(i\), and \(\alpha_1 + \alpha_2 = 1\).

- **(Conservation of momentum)**
  \[
  \frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{uu}) = -\nabla p + \nabla \cdot T + \rho g + f_{surf},
  \]
  \(T = \mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T) - \frac{2}{3} \mu (\nabla \cdot \mathbf{u}) \mathbf{I},
  \]
  \(\rho =\) mixture density = \(\rho_1 \alpha_1 + \rho_2 \alpha_2\),
  \(\mu =\) mixture viscosity.

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Governing Equations II

• (Continuum surface force)
  \[ f_{surf} = \gamma_K \nabla \alpha_1, \]
  \( K = \text{interface curvature} = -\nabla \cdot \left( \frac{\nabla \alpha_1}{|\nabla \alpha_1|} \right), \]
  \( \gamma = \text{surface tension}. \)

• (Material equation of state)
  \( \rho_1 = \text{water density} = \rho_0 + \varphi_1 \rho, \)
  \( \rho_2 = \text{air density} = \varphi_2 \rho. \)

• (Six degrees of freedom of motion)
  \( \sigma = -\rho I + T, \)
  \( F(t) = \text{force on object} = \int_{\partial \Omega(t)} \sigma \hat{n} \, dS, \)
  \( \tau(t) = \text{torque on object} = \int_{\partial \Omega(t)} r \times \sigma \hat{n} \, dS. \)

\[ \Rightarrow \text{Resulting velocity on object skin} \quad V(x, t) \]
  \[ \text{for} \quad x \in \partial \Omega(t) \]

• (Moving boundary condition on object skin)
  \[ u|_{\partial \Omega(t)} = V(x, t) \]
Scenario with 30°-Diving ($\theta = -30^\circ$, $\beta = 30^\circ$)

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Scenario with Nose-Diving ($\theta = -90^\circ$, $\beta = 93^\circ$)

Bending Moment

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